The See-Saw Mechanisms of Neutrino Mass Generation

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"The effort to strive for truth has to precede all other efforts"

Albert Einstein

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Kaushik Roy

Abstract

Neutrinos are ubiquitous in Nature as they are elusive. The flavor oscillation phenomenon detected in solar and atmospheric neutrinos provided conclusive evidence that neutrinos are massive. However, the origin and absolute masses of neutrinos are still unknown, since the neutrino oscillation parameters provide only the difference of the squared masses and the mixing angle between the different eigenstates. All experimental data point towards a spectrum where the neutrino masses are less than or of the order of 1 eV. This is an intriguing fact because the highly acclaimed and almost irrefutable framework of elementary particle interactions called the Standard Model fails to accommodate massive neutrinos. It is also revolutionary in the sense that these are the first particle interactions that point towards physics beyond the Standard Model.

In this thesis work, I briefly review one of the simplest and most elegant ways of explaining the observed smallness of neutrino masses; the see-saw mechanism. The fundamental idea is to augment the Standard Model particle content with new heavy particles. The see-saw mechanism provides a *natural* explanation of the tininess of neutrino masses in the sense that the masses are generated at a scale Λ which is higher than the electroweak scale and whose low-energy manifestation is through a set of effective higher dimension ($d > 4$) operators suppressed by appropriate powers of Λ. After an introduction to the fundamental concepts of neutrino masses in chapter 1, I elucidate in detail the theoretical framework of the three possible realizations of the see-saw mechanism, namely Types I, II and III throughout the chapters 2,3 and 4. Chapter 5 is a review of the various possible multi-lepton signatures of the see-saw mechanisms at the Large Hadron Collider (LHC) obtained from simulation studies. The concluding part consists of constructing a model having an extra SU(2) scalar doublet added to the particle content of the Type III see-saw picture and working out the various kinds of interactions possible in such a scenario.

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Chapter 1

Introduction

The introduction of the ‘neutrino’ into the particle-zoo by Wolfgang Pauli in the year 1930 [1] was a stroke of serendipity. Since then, decades of experimental and theoretical work of consummate elegance elevated the status of neutrino to an inseparable constituent of the accepted quantum description of fundamental particles and forces; the Standard Model of particle physics. Unlike other elementary fermions, the Standard Model predicts a zero mass for neutrinos, or equivalently only the left-chiral component is incorporated by the symmetries of the Standard Model. However, the discovery of neutrino oscillations [2,3] provided conclusive evidence of the existence of a small (but non-zero) neutrino mass. The origin of this small neutrino mass is still a mystery. An appealing mechanism for explaining this tininess of neutrino mass is the so-called see-saw mechanism, which can be realized in three possible ways. I discuss them sequentially in the coming chapters, but before that I introduce the concepts essential to understanding and appreciating the mechanism.

1.1 Dirac and Majorana masses

1.1.1 Dirac mass

The Dirac Lagrangian is given by

$$\mathcal{L}_{Dirac} = \overline{\psi}(x)(i\partial\!\!\!/ - m)\psi(x) \tag{1.1}$$

In terms of chiral components, $\psi = \psi_L + \psi_R$ where

$$\psi_L = L\psi = \frac{1-\gamma^5}{2}\psi$$

$$\psi_R = R\psi = \frac{1+\gamma^5}{2}\psi$$

The above equation becomes

$$\mathcal{L}_{Dirac} = (\overline{\psi}_L i\partial\!\!\!/\psi_L + \overline{\psi}_R i\partial\!\!\!/\psi_R) - m(\overline{\psi}_R\psi_L + \overline{\psi}_L\psi_R) \tag{1.2}$$

where I have made use of the properties of the chiral projection operators L and R

$$LR = 0 = RL, \qquad L^2 = L, \qquad R^2 = R$$

The Dirac mass-term given by

$$\mathcal{L}^{mass}_{Dirac} = -m(\overline{\psi}_R\psi_L + \overline{\psi}_L\psi_R) \tag{1.3}$$

couples left and right-handed components of the Dirac field. In the framework of the Standard Model, there are no right-handed components of neutrino thereby giving it the status of a massless lepton.

1.1.2 Majorana mass

In the above discussion, it was assumed that ψ_L and ψ_R are independent components. The field equations obtained from the Lagrangian given by eqn. (1.2) under such conditions are

$$i\partial\!\!\!/\psi_L = m\psi_R \tag{1.4}$$

$$i\partial\!\!\!/\psi_R = m\psi_L \tag{1.5}$$

In the year 1937, Ettore Majorana showed [4] that it is possible to represent a massive fermion field as a two-component spinor, by assuming that the right and left-handed components ψ_L and ψ_R are not independent. In fact, the two equations above are just two different ways of writing the same equation for one independent field, say ψ_L.

Taking the Hermitian conjugate of eqn. (1.5) and multiplying on the right by γ^0, obtain

$$-i\partial_\mu\overline{\psi}_R\gamma^\mu = m\overline{\psi}_L$$

Now, taking the transpose of this equation and multiplying on the left by the charge conjugation matrix $\mathcal{C}$, we get

$$i\partial_\mu\mathcal{C}\overline{\psi}^T_R = m\mathcal{C}\overline{\psi}^T_L \tag{1.6}$$

This is identical with eqn. (1.4) if we set

$$\psi_R = \xi \mathcal{C} \overline{\psi}_L^T \tag{1.7}$$

where ξ is an arbitrary phase factor ($| \xi |^2 = 1$). It can be eliminated by rephasing the field ψ_L as

$$\psi_L \rightarrow \xi^{1/2} \psi_L$$

Eqn. (1.7) gives the Majorana relation between ψ_R and ψ_L, which is totally valid because $\mathcal{C}\overline{\psi}_L^T$ is right-handed (can be shown directly by operating L on it). The Majorana condition for the field ψ is

$$\psi = \psi_L + \psi_R = \psi_L + \mathcal{C}\overline{\psi}_L^T = \psi^C = \mathcal{C}\overline{\psi}^T \tag{1.8}$$

A fermion that has the above property is its own charge conjugate, or in other words, its own antiparticle. Such a fermion must by default be electrically neutral, which can also be explicitly shown as is done below.

$$\begin{aligned} j^\mu = \overline{\psi}\gamma^\mu\psi &= \overline{\psi^C}\gamma^\mu\psi^C \\ &= -\psi^T \mathcal{C}^\dagger \gamma^\mu \mathcal{C} \overline{\psi}^T \\ &= \psi^T (\gamma^\mu)^T \overline{\psi}^T, (\text{since } \mathcal{C}^{-1}\gamma^\mu\mathcal{C} = -(\gamma^\mu)^T) \\ &= -\overline{\psi}\gamma^\mu\psi, (\text{using anticommutation of fermion fields}) \\ &= -j^\mu \Rightarrow j^\mu = 0 \end{aligned} \tag{1.9}$$

A neutrino therefore has the possibility of being a Majorana fermion.

The full Majorana Lagrangian can be written as

$$\mathcal{L}_{Majorana} = \frac{1}{2}\left[\overline{\psi}_L i\partial\!\!\!/ \psi_L + \overline{\psi_L^C} i\partial\!\!\!/ \psi_L^C - m(\overline{\psi_L^C}\psi_L + \overline{\psi_L}\psi_L^C)\right] \tag{1.10}$$

where the overall factor of $\frac{1}{2}$ is introduced to avoid double counting as ψ_L^C and $\overline{\psi_L}$ are not independent ($\psi_L^C = \mathcal{C}\overline{\psi_L^T}$).

Using the property of the charge-conjugation operator

$$\mathcal{C}^{-1}\gamma^\mu\mathcal{C} = -(\gamma^\mu)^T \tag{1.11}$$

The Majorana mass-term may be written as

$$\mathcal{L}_{Majorana}^{mass} = \frac{1}{2} m \psi_L^T \mathcal{C}^\dagger \psi_L + h.c \tag{1.12}$$

In the coming sections, I discuss in brief the attempt to generate neutrino mass in the Standard Model and some properties of the Majorana neutrino mass matrix.

1.2 Neutrino mass in a minimal extension of the Standard Model

1.2.1 Dirac neutrinos

Neutrino masses can be explained in the Standard Model by merely extending the leptonic sector to include three right-handed neutrinos $\nu_{\alpha R}$ ($\alpha = e, \mu, \tau$). The symmetries of the Standard Model do not restrict the number of these neutrinos but empirically addition of atleast two right-handed neutrinos can explain the observed data from neutrino oscillations. These right-handed neutrinos are called *sterile* because they do not take part in weak interactions (as well as strong and electromagnetic interactions like all neutrino fields). Their only interaction is gravitational. They are singlets of $SU(3)_C \otimes SU(2)_L$ and have weak-hypercharge, $Y = 0$. The Higgs-Lepton Yukawa Lagrangian now becomes

$$\mathcal{L}_{H,L} = -\left(\sum_{\alpha,\beta=e,\mu,\tau} Y'^{\ell}_{\alpha\beta}\overline{L_{\alpha L}}\Phi\ell'_{\beta R} + \sum_{\alpha,\beta=e,\mu,\tau} Y'^{\nu}_{\alpha\beta}\overline{L_{\alpha L}}\widetilde{\Phi}\nu'_{\beta R}\right) + h.c \tag{1.13}$$

where Y'^{ℓ} and Y'^{ν} are matrices of Yukawa couplings and

$$L_{\alpha L} = \begin{pmatrix} \nu'_{\alpha L} \\ \ell'_{\alpha L} \end{pmatrix}, \qquad \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \qquad \widetilde{\Phi} = i\tau_2\Phi^* \tag{1.14}$$

are the left-handed lepton doublets ($Y = -1$), the standard Higgs doublet ($Y = +1$) and the transformed Higgs doublet ($Y = -1$) respectively. In the unitary gauge, the Lagrangian given by eqn. (1.13) can be written after diagonalization as

$$\begin{aligned} \mathcal{L}_{H,L} &= -\left(\frac{v + h(x)}{\sqrt{2}}\right)\left[\overline{\ell_L}Y^{\ell}\ell_R + \overline{\nu_L}Y^{\nu}\nu_R\right] + h.c \\ &= -\left(\frac{v + h(x)}{\sqrt{2}}\right)\left[\sum_{\alpha=e,\mu,\tau} y^{\ell}_{\alpha}\overline{\ell_{\alpha R}}\ell_{\alpha R} + \sum_{\alpha=e,\mu,\tau} y^{\nu}_{\alpha}\overline{\nu_{\alpha L}}\nu_{\alpha R}\right] + h.c \end{aligned} \tag{1.15}$$

where $h(x)$ is the physical Higgs-field.

Therefore, the neutrino masses are given by

$$m_\alpha = \frac{y_\alpha^\nu v}{\sqrt{2}} \qquad (\alpha = e, \mu, \tau) \tag{1.16}$$

The neutrino masses obtained in this way are proportional to the Higgs Vacuum-Expectation-Value (VEV) v, similar to masses of charged leptons and quarks. There is however no explanation for the smallness of the parameters y_α^ν that should account for the small neutrino masses, relative to other constituents.

1.2.2 Majorana neutrinos

The Majorana mass term in eqn. (1.12) involves only the left-chiral field ν_L, which is present in the Standard Model. So, one might be tempted to think that a Majorana mass for the neutrino might be generated through the standard Higgs mechanism. But such a term cannot be incorporated in the Standard Model because the product $\nu_L^T \mathcal{C}^\dagger \nu_L$ transforms under $SU(2)_L \otimes U(1)_Y$ as

$$\nu_L^T \mathcal{C}^\dagger \nu_L \sim (2,-1) \otimes (2,-1) = (1,-2) \oplus (3,-2) \tag{1.17}$$

Thus, $\nu_L^T \mathcal{C}^\dagger \nu_L$ has $Y = -2$. Now, the Standard Model does not contain any weak isospin triplet with $Y = 2$ which can be combined with $\nu_L^T \mathcal{C}^\dagger \nu_L$ to produce a term invariant under the symmetries. So, a Majorana mass term cannot be accommodated in the Standard Model.

Working in the one generation picture, the lowest dimensional term which can be constructed with the Standard Model fields (respecting its symmetries) to generate a Majorana mass term is

$$\mathcal{L}_5 = \frac{g}{\mathcal{M}} (L_L^T \tau_2 \Phi) \mathcal{C}^\dagger (\Phi^T \tau L_L) + h.c \tag{1.18}$$

where g is a dimensionless coupling constant and $\mathcal{M}$ is a constant with dimensions of mass. The subscript 5 in the Lagrangian is due to the fact that it contains a product of fields of mass dimension five. As a consequence of electroweak symmetry breaking

$$\begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix} \xrightarrow[\text{Breaking}]{\text{Symmetry}} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \tag{1.19}$$

$\mathcal{L}_5$ generates the mass term

$$\mathcal{L}_{mass} = \frac{1}{2} \frac{g v^2}{\mathcal{M}} \nu_L^T \mathcal{C}^\dagger \nu_L + h.c \tag{1.20}$$

By comparison with eqn. (11), it is clear that (1.20) corresponds to a Majorana mass term with mass

$$m = \frac{gv^2}{\mathcal{M}} \tag{1.21}$$

The Lagrangian $\mathcal{L}_5$ is not acceptable in the framework of the Standard Model since it contains a product of terms of mass dimension five, which is not renormalizable. However, the Standard Model does not qualify as the final theory of everything, but is considered as an effective low-energy theory. Hence, it is likely that there are effective low-energy Lagrangian terms which respect the symmetries of the Standard Model, but are non-renormalizable.

The important thing to note here is that the Majorana mass generated is proportional to the ratio $gv^2/\mathcal{M}$. Since v is the scale of electroweak symmetry breaking, it sets the scale of the Dirac fermion masses generated through the Higgs mechanism. Hence,

$$m \propto \frac{m_D^2}{\mathcal{M}} \tag{1.22}$$

I will return to this relation when I discuss type I see-saw mechanism in chapter 2.

1.3 Majorana mass matrix

The one generation Majorana mass term is written as

$$\mathcal{L}^{mass}_{Majorana} = \frac{1}{2} m \nu_L^T \mathcal{C}^\dagger \nu_L + h.c \tag{1.23}$$

In the three generation picture, this mass term appears as a 3×3 matrix. Defining the array of left-handed flavor neutrino fields

$$\boldsymbol{\nu}'_L = \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \tag{1.24}$$

we can construct the Majorana mass term

$$\begin{aligned} \mathcal{L}^{mass}_{Majorana} &= \frac{1}{2} \boldsymbol{\nu}_L'^T \mathcal{C}^\dagger M^L \boldsymbol{\nu}'_L + h.c \\ &= \frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu_{\alpha L}'^T \mathcal{C}^\dagger M^L_{\alpha\beta} \nu'_{\beta L} + h.c \end{aligned} \tag{1.25}$$

In general, the mass matrix M^L is a complex symmetric matrix since

$$\begin{aligned}\sum_{\alpha,\beta} \nu'^T_{\alpha L}\mathcal{C}^\dagger M^L_{\alpha\beta}\nu'_{\beta L} &= -\sum_{\alpha,\beta} \nu'^T_{\beta L}M^L_{\alpha\beta}(\mathcal{C}^\dagger)^T\nu'_{\alpha L} \\ &= \sum_{\alpha,\beta} \nu'^T_{\beta L}M^L_{\alpha\beta}\mathcal{C}^\dagger\nu'_{\alpha L} \\ &= \sum_{\alpha,\beta} \nu'^T_{\alpha L}\mathcal{C}^\dagger M^L_{\beta\alpha}\nu'_{\beta L}\end{aligned} \tag{1.26}$$

The Lagrangian being a number, we can always set each of the terms to be equal to its transpose. The negative sign in the first line is due to anticommutation property of fermion fields and I have used the $\mathcal{C} = -\mathcal{C}^T$ in the second line. It is evident from eqns. (1.26) that

$$M^L_{\alpha\beta} = M^L_{\beta\alpha} \tag{1.27}$$

thereby showing that M^L is symmetric. Diagonalizing this matrix with a transformation,

$$(V^\nu_L)^T M^L V^\nu_L = M, \qquad M_{kj} = m_k\delta_{kj} \tag{1.28}$$

where

$$\boldsymbol{\nu}'_L = V^\nu_L \boldsymbol{n}_L = V^\nu_L \begin{pmatrix}\nu_{1L}\\ \nu_{2L}\\ \nu_{3L}\end{pmatrix} \tag{1.29}$$

The three-generation Majorana mass term thus becomes

$$\begin{aligned}\mathcal{L}^{mass}_{Majorana} &= \frac{1}{2}\boldsymbol{n}^T_L\mathcal{C}^\dagger M\boldsymbol{n}_L + h.c \\ &= \frac{1}{2}\sum_{k=1}^{3} m_k\nu^T_{kL}\mathcal{C}^\dagger\nu_{kL} + h.c\end{aligned} \tag{1.30}$$

Chapter 2

Type I see-saw mechanism

It is evident from the arguments made in the previous chapter that we need a different model to explain the smallness of the neutrino masses. Such a model was proposed in 1979 by Murray Gell-Mann, Pierre Ramond and Richard Slansky working in the U.S., and independently by Tsutomu Yanagida of Tokyo University. This is the Type I *see-saw* mechanism [5], whose idea is to introduce Majorana-type right-handed neutrinos in the Standard Model with very heavy mass, possibly at a scale where the three forces of the Standard Model unify. The Heisenberg Uncertainty Principle, which allows energy conservation to be violated on small time-intervals, allows a left-handed neutrino to spontaneously convert into a heavy right-handed neutrino for a very brief moment before reverting back to being a left-handed neutrino. This results in a very small observed Majorana mass for the left-handed neutrino, the smallness being attributed to the heaviness of the right-handed neutrino.

2.1 One generation Dirac-Majorana mass term

It is clear that the chiral fields ν_L and ν_R are the building blocks of the neutrino Lagrangian. The chiral field ν_L exists, because it is present in the Standard Model and enters the charged-current weak interaction Lagrangian. However, it is not known whether ν_R exits, but it is allowed by the Standard Model symmetries. If only ν_L exists, then the neutrino Lagrangian can contain only the Majorana mass-term

$$\mathcal{L}^L_{mass} = \frac{1}{2} m_L \nu_L^T \mathcal{C}^\dagger \nu_L + h.c \tag{2.1}$$

and the neutrino is a Majorana particle. If ν_R also exists, the neutrino Lagrangian can contain the Dirac mass-term

$$\mathcal{L}^{D}_{mass} = -m_D \overline{\nu_R} \nu_L + h.c \tag{2.2}$$

In addition, the neutrino Lagrangian can also contain the Majorana mass-term for ν_L and the Majorana mass-term for ν_R.

$$\mathcal{L}^{R}_{mass} = \frac{1}{2} m_R \nu_R^T \mathcal{C}^\dagger \nu_R + h.c \tag{2.3}$$

Thus it is possible to have the Dirac-Majorana neutrino mass term

$$\begin{aligned} \mathcal{L}^{D+M}_{mass} &= \mathcal{L}^{D}_{mass} + \mathcal{L}^{L}_{mass} + \mathcal{L}^{R}_{mass} \\ &= -m_D(\overline{\nu_R}\nu_L + h.c) + \frac{1}{2} m_L(\nu_L^T \mathcal{C}^\dagger \nu_L + h.c) + \frac{1}{2} m_R(\nu_R^T \mathcal{C}^\dagger \nu_R + h.c) \end{aligned} \tag{2.4}$$

This is a unique feature of neutrinos that they can have both the left and right-handed Majorana terms; owing to their zero electric charge unlike the other fermions of the Standard Model. As already discussed in subsection 1.2.2, the Majorana mass-term is not allowed by the Standard Model symmetries but the mass-term for ν_R does not have any such issues. So, the Dirac-Majorana mass term in eqn. (2.4) with $m_L = 0$ is allowed in the framework of the Standard Model, with the only addition of the right-chiral field ν_R.

Let us introduce the following column matrix of left-handed chiral fields

$$\boldsymbol{N}_L = \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix} = \begin{pmatrix} \nu_L \\ \mathcal{C}\overline{\nu_R}^T \end{pmatrix} \tag{2.5}$$

Eqn. (2.4) can now be re-written as

$$\mathcal{L}^{D+M}_{mass} = \frac{1}{2} \boldsymbol{N}_L^T \mathcal{C}^\dagger M \boldsymbol{N}_L + h.c \tag{2.6}$$

with the symmetric mass matrix

$$M = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \tag{2.7}$$

For getting definite masses, this matrix needs to be diagonalized. This is achieved through the unitary matrix

$$U = \mathcal{O}\rho$$

where $\mathcal{O}$ is an orthogonal 2×2 matrix and ρ is a diagonal metrix of phases

$$\mathcal{O} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \qquad \rho = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} \tag{2.8}$$

The matrix $\mathcal{O}$ is chosen so as to diagonalize the mass-matrix in eqn. (2.7).

$$\mathcal{O}^T M \mathcal{O} = \begin{pmatrix} m_1' & 0 \\ 0 & m_2' \end{pmatrix} \tag{2.9}$$

where m_1' and m_2' are the eigenvalues of the mass-matrix (taking $m_L = 0$ now)

$$m_1' = \frac{1}{2}\left[m_R - \sqrt{m_R^2 + 4m_D^2}\right] \tag{2.10}$$

$$m_2' = \frac{1}{2}\left[m_R + \sqrt{m_R^2 + 4m_D^2}\right] \tag{2.11}$$

When $m_D^2 > 0$, then m_1' is negative. The role of the matrix ρ is to change the sign of the first mass eigenvalue if $m_1' < 0$ through

$$U^T M U = \rho^T \mathcal{O}^T M \mathcal{O} \rho = \begin{pmatrix} \rho_1^2 m_1' & 0 \\ 0 & \rho_2^2 m_2' \end{pmatrix} \tag{2.12}$$

The real and positive eigenvalues are given by $m_k = \rho_k^2 m_k'$ with $\rho_2^2 = 1$, whereas $\rho_1^2 = -1$ if $m_1' < 0$ and $\rho_1^2 = 1$ if $m_1' > 0$.

The massive neutrino fields obtained post-diagonalization

$$\boldsymbol{N}_L = U \boldsymbol{n}_L = U \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix} \tag{2.13}$$

when replaced in eqn. (2.6) yields

$$\mathcal{L}_{mass}^{D+M} = \frac{1}{2} \sum_{k=1,2} m_k \nu_{kL}^T \mathcal{C}^\dagger \nu_{kL} + h.c = -\frac{1}{2} \sum_{k=1,2} m_k \overline{\nu_k} \nu_k \tag{2.14}$$

where $\nu_k = \nu_{kL} + \nu_{kL}^C$. This implies that the massive neutrinos are Majorana particles.

Now coming to the case of our interest,

$$m_D \ll m_R, \qquad m_L = 0 \tag{2.15}$$

From eqns. (2.10) and (2.11), we get

$$m_1' \simeq -\frac{m_D^2}{m_R}, \qquad m_2' \simeq m_R \tag{2.16}$$

m_1' being negative, $\rho_1^2 = -1$ and so

$$m_1 \simeq \frac{m_D^2}{m_R}, \qquad m_2 \simeq m_R \tag{2.17}$$

This means that ν_2 is as heavy as m_R and ν_1 is very light; its mass being suppressed by the small ratio m_D/m_R. This is the so-called Type I *see-saw* mechanism; the nomenclature being due to the fact that the heavy mass of ν_2 (going down of one side of a see-saw) is responsible for the lightness of ν_1 (going up of the other end).

From eqn. (2.5), we get the mixing relations

$$\nu_L = U_{11}\nu_{1L} + U_{12}\nu_{2L} \tag{2.18}$$

$$\nu_R^C = U_{21}\nu_{1L} + U_{22}\nu_{2L} \tag{2.19}$$

The mixing matrix obtained in this case is

$$U = \begin{pmatrix} i\cos\theta & \sin\theta \\ -i\sin\theta & \cos\theta \end{pmatrix} \tag{2.20}$$

So

$$\nu_L = i\cos\theta\nu_{1L} + \sin\theta\nu_{2L} \tag{2.21}$$

$$\nu_R^C = -i\sin\theta\nu_{1L} + \cos\theta\nu_{2L} \tag{2.22}$$

The mixing angle θ can be obtained from the eigen-value equations as

$$\tan(2\theta) = \frac{2m_D}{m_R} \tag{2.23}$$

In type I see-saw, the mixing angle is very small (since $m_R >> m_D$), so that

$$\nu_{1L} \simeq -i\nu_L, \qquad \nu_{2L} \simeq \nu_R^C \tag{2.24}$$

Thus, ν_1(light) is composed mainly of active ν_L and ν_2(heavy) is composed mainly of sterile ν_R.

Table 1: Masses generated by type I see-saw mechanism.

m_{light}	m_D	m_R
0.1 eV	1 Mev	10^4 GeV
0.1 eV	1 GeV	10^{10} GeV
0.1 eV	100 Gev	10^{14} GeV
0.01 eV	1 Mev	10^5 GeV
0.01 eV	1 GeV	10^{11} GeV
0.01 eV	100 GeV	10^{15} GeV

Thus, the scale of m_R should be very high, close to the grand unification order of $10^{14} - 10^{16}$ GeV for explaining the small neutrino masses.

2.2 Integrating out the heavy field

The type I see-saw mechanism can be thought of as a particular case of the general effective dimension-5 operator $\mathcal{L}_5$ discussed at the end of subsection 1.2.2. Let us consider the Dirac-Majorana mass term given by eqn. (2.4) with $m_L = 0$.

$$\mathcal{L}^{D+M} = -m_D(\overline{\nu_R}\nu_L + \overline{\nu_L}\nu_R) + \frac{1}{2}m_R(\nu_R^T\mathcal{C}^\dagger\nu_R + \nu_R\mathcal{C}\nu_R^*) \tag{2.25}$$

Above the electroweak symmetry breaking scale, the symmetries of the Standard Model require that $\mathcal{L}^{D+M}$ be written as

$$\mathcal{L}^{D+M} = -y^\nu(\overline{\nu_R}\tilde{\Phi}^\dagger L_L + \overline{L}_L\tilde{\Phi}\nu_R) + \frac{1}{2}m_R(\nu_R^T\mathcal{C}^\dagger\nu_R + \nu_R\mathcal{C}\nu_R^*) \tag{2.26}$$

Below the electroweak symmetry breaking scale, the Dirac part of $\mathcal{L}^{D+M}$ generates the Dirac neutrino mass

$$m_D = \frac{y^\nu v}{\sqrt{2}} \tag{2.27}$$

If the mass m_R is very heavy at Standard Model energies, the right-handed chiral field ν_R can be integrated away by considering it in the static limit whereby the kinetic term in the Euler-Lagrange equations of motion for ν_R can be neglected.

$$\frac{\partial\mathcal{L}^{D+M}}{\partial\nu_R} = \partial_\mu\left(\frac{\partial\mathcal{L}^{D+M}}{\partial(\partial_\mu\nu_R)}\right) \simeq 0 \tag{2.28}$$

This is equivalent to

$$\nu_R^T \simeq \frac{y^\nu}{m_R}\overline{L}_L\tilde{\Phi}\mathcal{C} \tag{2.29}$$

$$\nu_R \simeq -\frac{y^\nu}{m_R}\tilde{\Phi}^T \mathcal{C}\overline{L}_L^T \tag{2.30}$$

Substituting this in eqn. (2.26), we get

$$\mathcal{L}^{D+M} = -\frac{1}{2}\frac{(y^\nu)^2}{m_R}\left[(L_L^T\tau_2\Phi)\mathcal{C}^\dagger(\Phi^T\tau_2 L_L) - (\overline{L}_L\tau_2\Phi^*)\mathcal{C}(\Phi^\dagger\tau_2\overline{L}_L^T)\right] \tag{2.31}$$

This coincides with the Lagrangian given by eqn. (1.8) if

$$g = -\frac{(y^\nu)^2}{2}, \qquad \mathcal{M} = m_R \tag{2.32}$$

Using the relations in (2.24),

$$\mathcal{L}_5^{D+M} \simeq \frac{1}{2}\frac{m_D^2}{m_R}\left[\nu_{1L}^T\mathcal{C}^\dagger\nu_{1L} + \nu_{1L}^\dagger\mathcal{C}\nu_{1L}^*\right] \tag{2.33}$$

This is a correct Majorana mass term for ν_{1L}, whose mass is given by the see-saw formula (2.17)

$$m_1 \simeq \frac{m_D^2}{m_R}$$

2.3 Generalizations

We can generalize the above picture to three-generation Dirac-Majorana neutrino mixing. In this picture, in addition to the three known *active* left-handed neutrino fields $\nu'_{eL}, \nu'_{\mu L}, \nu'_{\tau L}$, there can be arbitrarily many, N_s sterile neutrinos ν_{sR} (unprimed because they do not participate in weak interactions). So, the most general mass term is the Dirac-Majorana mass term

$$\mathcal{L}_{mass}^{D+M} = \mathcal{L}_{mass}^{D} + \mathcal{L}_{mass}^{L} + \mathcal{L}_{mass}^{R} \tag{2.34}$$

with

$$\mathcal{L}_{mass}^{L} = \frac{1}{2}\sum_{\alpha,\beta=e,\mu,\tau}\nu_{\alpha L}^{\prime T}\mathcal{C}^\dagger M_{\alpha\beta}^L\nu'_{\beta L} + h.c \tag{2.35}$$

$$\mathcal{L}_{mass}^{R} = \frac{1}{2}\sum_{s,s'=1,2,\ldots,N_s}\nu_{sR}^T\mathcal{C}^\dagger M_{ss'}^R\nu_{s'R} + h.c \tag{2.36}$$

and the Dirac mass-term by

$$\mathcal{L}_{mass}^{D} = -\sum_{s,s'=1,2,\ldots,N_s}\sum_{\alpha,\beta=e,\mu,\tau}\overline{\nu_{sR}}M_{s\alpha}^D\nu'_{\alpha L} + h.c \tag{2.37}$$

Defining

$$\boldsymbol{N}_L^{'} = \begin{pmatrix} \boldsymbol{\nu}_L^{'} \\ \boldsymbol{\nu}_R^C \end{pmatrix} \tag{2.38}$$

where

$$\boldsymbol{\nu}_L^{'} = \begin{pmatrix} \nu_{eL}' \\ \nu_{\mu L}' \\ \nu_{\tau L}' \end{pmatrix} \qquad and \qquad \boldsymbol{\nu}_R^C = \begin{pmatrix} \nu_{1R}^C \\ \nu_{2R}^C \\ . \\ . \\ . \\ \nu_{N_sR}^C \end{pmatrix} \tag{2.39}$$

are the arrays of active and sterile neutrino fields. The total Lagrangian can now be written as

$$\mathcal{L}_{mass}^{D+M} = \frac{1}{2}\boldsymbol{N}_L^{'T} C^\dagger M^{D+M} \boldsymbol{N}_L^{'} + h.c \tag{2.40}$$

where the $N \times N$, $(N = 3 + N_s)$ symmetric mass matrix is given by

$$M^{D+M} = \begin{pmatrix} M^L & M^{D^T} \\ M^D & M^R \end{pmatrix} \tag{2.41}$$

Since M^L and M^R are symmetric Majorana mass matrices, so M^{D+M} is symmetric. To diagonalize, we use the unitary transformation

$$(V_L^\nu)^T M^{D+M} V_L^\nu = M \tag{2.42}$$

In the type I see-saw picture or when the eigenvalues of M^R are much larger than all the elements of M^D with $M^L = 0$, the matrix can be diagonalized in blocks with the ansatz [6]

$$V_L^\nu = e^{iH} V = e^{iH} \begin{pmatrix} V_1 & 0 \\ 0 & V_2 \end{pmatrix} \tag{2.43}$$

where $H = \begin{pmatrix} 0 & S \\ S' & 0 \end{pmatrix}$ with $S' = S^\dagger$ is a unitary matrix and V is a diagonal matrix of phases that fix the sign of the light neutrino mass eigenvalues generated after diagonalization. The justification that such an ansatz is valid is explained in Appendix A.

The unitary matrix V_L^ν upto corrections of the order $\mathcal{O}[(M^R)^{-1}M^D]$ is

$$V_L^\nu = \begin{pmatrix} 1 - \frac{1}{2}(M^D)^\dagger[M^R M^{R\dagger}]^{-1}M^D & (M^{R^{-1}}M^D)^\dagger \\ -M^{R^{-1}}M^D & 1 - \frac{1}{2}M^{R^{-1}}M^D M^{D\dagger}(M^{R^{-1}})^\dagger \end{pmatrix} \begin{pmatrix} V_1 & 0 \\ 0 & V_2 \end{pmatrix} \tag{2.44}$$

The resulting masses for the three active and N_s sterile neutrinos are given by

$$M_{light} \simeq V_1^T[-M^{D^T}(M^R)^{-1}M^D]V_1, \qquad M_{heavy} \simeq V_2^T M^R V_2 \tag{2.45}$$

Similar to the discussion in eqn. (2.12), the signs of the masses are adjusted by V_1 and V_2.

It is to be noted that atleast two sterile neutrinos are to be added to explain the observed data from neutrino oscillations. From eqn. (2.37), it is clear that M^D is an $N_s \times 3$ rectangular matrix for the general case of adding N_s sterile neutrinos. Hence, the mass relations (2.45) tell us that even if we add one sterile neutrino, we get a 3×3 light neutrino mass-matrix. Let us consider this simple scenario of adding a single right-handed neutrino ν_R. The 1×3 mass-matrix M^D can in general be written as

$$M^D = \begin{pmatrix} m_1 & m_2 & m_3 \end{pmatrix} \tag{2.46}$$

so that

$$M_{light} \simeq -\frac{1}{m_R}\begin{pmatrix} m_1^2 & m_1m_2 & m_1m_3 \\ m_2m_1 & m_2^2 & m_2m_3 \\ m_3m_1 & m_3m_2 & m_3^2 \end{pmatrix} \tag{2.47}$$

which is visibly symmetric and on diagonalization becomes

$$M_{light}^{diag} = -\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (m_1^2+m_2^2+m_3^2)/m_R \end{pmatrix} \tag{2.48}$$

Hence, we get only one massive neutrino. Through a similar process, it can be shown that by adding two right-handed neutrinos ν_{1R} and ν_{2R}, there are two massive light neutrinos on diagonalizing the mass matrix M_{light}.

From neutrino oscillation data, two kinds of mass-splitting Δm_{21}^2 and $\mid \Delta m_{31}^2 \mid$ are observed which implies that atleast two neutrinos should have non-zero masses. Hence, it is clear from the above demonstration that we require an addition of atleast two right-handed neutrinos for consistency with observed experimental data.

2.4 Weak Interactions

2.4.1 Coupling to W bosons

Let us begin by investigating the leptonic part of the charged-current weak interaction Lagrangian. The leptonic charged-current is given by

$$j^{\mu}_{W,L} = 2\overline{\boldsymbol{\nu}'_L}\gamma^{\mu}\boldsymbol{\ell}'_L \tag{2.49}$$

In the three-generation Dirac-Majorana mixing picture, this can be written in terms of mass eigenstates as

$$j^{\mu}_{W,L} = 2\overline{\boldsymbol{n}_L}\mathcal{U}^{\dagger}\gamma^{\mu}\boldsymbol{\ell}_L \tag{2.50}$$

where

$$\boldsymbol{\nu}'_L = \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} = K^{\nu}_L \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ . \\ . \\ . \\ \nu_{NL} \end{pmatrix} = K^{\nu}_L \boldsymbol{n}_L \tag{2.51}$$

where $\boldsymbol{\ell}'_L = V^{\ell}_L \ell_L$ and $\mathcal{U} = V^{\ell\dagger}_L K^{\nu}_L$ is a rectangular $3 \times N$ rectangular matrix with components

$$\mathcal{U}_{\alpha k} = \sum_{\beta=e,\mu,\tau} (V^{\ell\dagger}_L)_{\alpha\beta}(K^{\nu}_L)_{\beta k} \qquad k = 1, 2, ..., N \tag{2.52}$$

$\mathcal{U}$ is not unitary because although $\mathcal{U}\mathcal{U}^{\dagger} = 1$, $\mathcal{U}^{\dagger}\mathcal{U} \neq 1$. In terms of the left-handed flavor neutrino fields defined by $\boldsymbol{\nu}_L = \mathcal{U}\boldsymbol{n}_L$, the leptonic weak charged current can be written as

$$j^{\mu}_{W,L} = 2\overline{\boldsymbol{\nu}_L}\gamma^{\mu}\boldsymbol{\ell}_L = 2\sum_{\alpha=e,\mu,\tau} \overline{\nu_{\alpha L}}\gamma^{\mu}\ell_{\alpha L} \tag{2.53}$$

The mixing of active and sterile neutrinos is given by

$$\nu_{\alpha L} = \sum_{k=1}^{N} \mathcal{U}_{\alpha k}\nu_{kL} \qquad (\alpha = e, \mu, \tau) \tag{2.54}$$

$$\nu^{C}_{sR} = \sum_{k=1}^{N} (V^{\nu}_L)_{sk}\nu_{kL} \qquad (s = 1, 2, ..., N_s) \tag{2.55}$$

Equations (2.54) and (2.55) show that the active and sterile neutrino states are superpositions of the same massive neutrino fields and hence there can be oscillations between the two. This can also be seen by noticing that the non-unitarity of the mixing matrix $\mathcal{U}$ implies that the total probability of the active flavors is not conserved.

2.4.2 Coupling to Z bosons

The leptonic weak-neutral current is given by

$$\begin{aligned} j^{\mu}_{Z,L} &= \overline{\boldsymbol{\nu}_L}\gamma^{\mu}\boldsymbol{\nu}_L \\ &= \frac{1}{2}\overline{\boldsymbol{\nu}}\gamma^{\mu}(1-\gamma^5)\boldsymbol{\nu} \end{aligned} \tag{2.56}$$

which is of the V-A form. However, if $\boldsymbol{\nu}$ represents a Majorana field, then the vector part of this current given by, $\overline{\boldsymbol{\nu}}\gamma^{\mu}\boldsymbol{\nu}$, vanishes by virtue of the Majorana condition as shown in eqn. (1.9). Hence, the current can be written as

$$j^{\mu}_{Z,L} = -\frac{1}{2}\overline{\boldsymbol{\nu}}\gamma^{\mu}\gamma^5\boldsymbol{\nu} \tag{2.57}$$

Thus, Majorana neutrinos have a pure axial neutral current interaction, unlike Dirac neutrinos. This difference raises the hope [7] of distinguishing Dirac and Majorana neutrinos in neutral current reactions. Unfortunately, the relevant quantities in such processes like scattering cross-sections, differ only by a term proportional to the neutrino mass, thereby making it almost impossible to make such a distinction.

In the type I see-saw picture, this neutral current can be written as

$$\begin{aligned} j^{\mu}_{Z,L} &= \overline{\boldsymbol{\nu}'_L}\gamma^{\mu}\boldsymbol{\nu}'_L \\ &= \sum_{\alpha=e,\mu,\tau}\overline{\nu'_{\alpha L}}\gamma^{\mu}\nu'_{\alpha L} \\ &= \overline{\boldsymbol{n}_L}(K^{\nu\dagger}_L K^{\nu}_L)\gamma^{\mu}\boldsymbol{n}_L \end{aligned} \tag{2.58}$$

From eqn. (2.52), we get $\mathcal{U}^{\dagger}\mathcal{U}$=$K^{\nu\dagger}_L K^{\nu}_L$.

Now,

$$\begin{aligned} \boldsymbol{N}'_L &= V^{\nu}_L\boldsymbol{n}_L \\ \Rightarrow \begin{pmatrix}\boldsymbol{\nu}'_L \\ \boldsymbol{\nu}^C_R\end{pmatrix} &= \begin{pmatrix} V_a & V_b \\ V_c & V_d\end{pmatrix}\begin{pmatrix}\boldsymbol{\nu}_{light} \\ \boldsymbol{\nu}_{heavy}\end{pmatrix} \end{aligned} \tag{2.59}$$

So

$$K^{\nu\dagger}_L K^{\nu}_L = \begin{pmatrix} V^{\dagger}_a \\ V^{\dagger}_b\end{pmatrix}\begin{pmatrix} V_a & V_b\end{pmatrix} \tag{2.60}$$

In the type I see-saw picture, we can use eqn. (2.44) to write the weak-neutral current as

$$j^{\mu}_{Z,L} = \overline{\boldsymbol{n}_L} P \gamma^{\mu} \boldsymbol{n}_L \tag{2.61}$$

where

$$\begin{aligned} P &= K_L^{\nu\dagger} K_L^{\nu} \\ &= \begin{pmatrix} 1 - V_1^{\dagger} M^{D\dagger} (M^{R\dagger})^{-1} M^{R^{-1}} M^D V_1 & V_1^{\dagger} M^{D\dagger} (M^{R\dagger})^{-1} V_2 \\ V_2^{\dagger} M^{R^{-1}} M^D V_1 & V_2^{\dagger} M^{R^{-1}} M^D M^{D\dagger} (M^{R\dagger})^{-1} V_2 \end{pmatrix} \end{aligned} \tag{2.62}$$

The interesting thing to note from eqn. (2.61) is that the neutrino current is not diagonal in the massive fields. So, the Flavor-Changing-Neutral-Currents (FCNC's) are not suppressed in the general Dirac-Majorana case unlike the Standard Model scenario where they are suppressed by the Glashow–Iliopoulos–Maiani (GIM) mechanism. So, it is possible to have neutral-current transitions among different massive neutrinos.

Chapter 3

Type II see-saw mechanism

In the type I see-saw mechanism, an attempt to explain the smallness of neutrino masses is made by adding right-handed neutrinos to the particle content of the Standard Model. However, it is seen that the usual realization of such a mechanism requires the mass scale m_R of the right-handed neutrinos ν_R to be of the order of the GUT scale ($10^{14} - 10^{16}$ GeV) and hence the possibility of direct tests of the type I see-saw mechanism seems remote.

Another mechanism that attempts to explain the smallness of neutrino masses is the type II see-saw mechanism [8] whose basic idea is to augment the scalar sector of the Standard Model by including a colorless scalar field Δ that transforms as a triplet under the $SU(2)_L$ gauge group with weak-hypercharge $Y = 2$. A Majorana neutrino mass is generated when the neutral component of the Higgs triplet acquires a VEV and the smallness of this mass is explained by the heavy nature of the triplet Higgs as well as its suppressed VEV. This will be elaborated in the sections that follow.

3.1 Yukawa couplings and the enlarged scalar sector

In the electroweak sector, the lepton fields and Higgs scalars have the following transformation properties under $SU(2)_L \times U(1)_Y$

$$\begin{aligned} L_{\alpha L} &= \begin{pmatrix} \nu'_{\alpha L} \\ \ell'_{\alpha L} \end{pmatrix} \sim (2,-1) \qquad \alpha = e, \mu, \tau \\ \ell'_{\alpha R} &= e'_R, \mu'_R, \tau'_R \sim (1,-2) \\ \Phi &= \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (2,+1) \end{aligned} \tag{3.1}$$

Terms bilinear in the leptonic fields are given by

$$\begin{aligned} \overline{L_{\alpha L}} \ell'_R &\sim (2,+1) \otimes (1,-2) = (2,-1) \\ \overline{L_L^C} L_L &\sim (2,-1) \otimes (2,-1) = (1,-2) \oplus (3,-2) \\ \overline{\ell_R^C} \ell_R &\sim (1,-2) \otimes (1,-2) = (1,-4) \end{aligned} \tag{3.2}$$

The first kind of term can be multiplied with a standard Higgs doublet, whose transformation is given in eqn. (3.1), to produce a $SU(2)_L \otimes U(1)_L$ gauge-invariant term. This is the normal Yukawa term that appears in the Standard Model.

However, the Standard Model may be enlarged by introducing additional scalars which can form Yukawa couplings that respect the symmetries of the Model. Three types of possibilities arise:

(1)Triplet : $\boldsymbol{\xi} \sim (3,+2)$

(2)Singly charged singlet : $\mathcal{H}^+ \sim (1,+2)$

(3)Doubly charged singlet : $\mathcal{H}^{++} \sim (1,+4)$

The first scenario of adding a Higgs triplet is of relevance to us. This idea of adding a scalar triplet to the Standard Model was first mentioned in a work [9] by W. Konetschny and W. Kummer in 1977. In their work, they showed that additional scalar triplets S^+,S^{++} and a scalar triplet $\Delta = (\Delta^{++}, \Delta^+, \Delta^0)$ permit Yukawa couplings, thereby allowing lepton-flavor violating transitions like $\mu \to e\gamma$ and $\mu \to 3e$. In a later work [10] by G. B Gelmini and M. Roncadelli in 1981, the same idea of an additional scalar triplet was used but the triplet was assigned a lepton number. A triplet VEV would break lepton number spontaneously thus leading to a massless, weakly interacting Goldstone boson called the *Majoron*. Since it would contribute unacceptably to the invisible decay-width of the Z-boson, such a 'triplet Majoron' model has been disfavored by Z-factory data.

3.2 The model

Besides the usual Standard Model particle content, this model requires the existence of a $Y = 2$, $SU(2)_L$ triplet $\boldsymbol{\xi}$.

Given a

$$\boldsymbol{\xi} = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix} \qquad \text{with Y=+2} \tag{3.3}$$

the 2×2 matrix representation of $\boldsymbol{\xi}$ is

$$\Delta = \frac{\boldsymbol{\tau}.\boldsymbol{\xi}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{pmatrix} \xi_3 & \xi_1 - i\xi_2 \\ \xi_1 + i\xi_2 & -\xi_3 \end{pmatrix} \tag{3.4}$$

which transforms under $SU(2)_L$ as $\Delta = \mathcal{U}(x)\Delta\mathcal{U}^\dagger(x)$, where $\mathcal{U}(x) = e^{i\theta^a(x)I^a}$ and $I^a = \frac{\tau^a}{2}$ (a=1,2,3) are the generators of the weak isospin group $SU(2)_L$. Using the Gell-Mann-Nishijima relation $Q = I_3 + \frac{Y}{2}$, we can write Δ in terms of the electric charge eigenstates as

$$\Delta = \frac{1}{\sqrt{2}} \begin{pmatrix} \delta^+ & \sqrt{2}\delta^{++} \\ \sqrt{2}\delta^0 & -\delta^+ \end{pmatrix} = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix} \tag{3.5}$$

where the electric charge eigenstates are defined by

$$\begin{aligned} \delta^{++} &= \frac{1}{\sqrt{2}}(\xi_1 - i\xi_2) \\ \delta^+ &= \xi_3, \quad \delta^0 = \frac{1}{\sqrt{2}}(\xi_1 + i\xi_2) \end{aligned} \tag{3.6}$$

The triplet $\boldsymbol{\xi}$ in terms of the electric charge eigenstates is

$$\boldsymbol{\xi} = \begin{pmatrix} (\delta^0 + \delta^{++})/\sqrt{2} \\ i(\delta^{++} - \delta^0)/\sqrt{2} \\ \delta^+ \end{pmatrix} \tag{3.7}$$

The electric charge assignments for the upper and lower component fields as given in eqns. (3.1) and (3.5) are conventions and can be interchanged by taking $Y_\Delta = -2$ and $Y_\Phi = -1$, thereby causing an exchange of the upper and lower components of the fermion weak doublets, without affecting the physical content.

The renormalizable, gauge-invariant Lagrangian representing the scalar sector is given by [11]

$$\mathcal{L} = (\mathcal{D}_\mu\Phi)^\dagger(\mathcal{D}^\mu\Phi) + Tr(\mathcal{D}_\mu\Delta)^\dagger(\mathcal{D}^\mu\Delta) - V(\Phi,\Delta) + \mathcal{L}_{Yukawa} \tag{3.8}$$

where

$$\mathcal{D}_\mu\Phi = \partial_\mu\Phi + igI^a A^a_\mu\Phi + i\frac{g'}{2}B_\mu\Phi \tag{3.9}$$

$$\mathcal{D}_\mu\Delta = \partial_\mu\Delta + ig[I^a A^a_\mu, \Delta] + ig'B_\mu\Delta \tag{3.10}$$

(A^a_μ, g) and (B_μ, g') are $SU(2)_L$ and $U(1)_Y$ gauge fields and couplings respectively. The potential $V(\Phi, \Delta)$ is given by

$$\begin{aligned} V(\Phi,\Delta) =& -m_H^2\Phi^\dagger\Phi + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2 + M_\Delta^2 Tr(\Delta^\dagger\Delta) + [\mu(\Phi^T i\tau_2\Delta^\dagger\Phi) + h.c] \\ &+ \lambda_1(\Phi^\dagger\Phi)Tr(\Delta^\dagger\Delta) + \lambda_2(Tr(\Delta^\dagger\Delta))^2 + \lambda_3 Tr(\Delta^\dagger\Delta)^2 + \lambda_4\Phi^\dagger\Delta\Delta^\dagger\Phi \end{aligned} \tag{3.11}$$

Here Tr denotes trace over 2×2 matrices. $\mathcal{L}_{Yukawa}$ contains the Yukawa sector of the Standard Model alongwith an added term which after spontaneous symmetry breaking will lead to the desired Majorana mass terms for the neutrinos, without the requirement of right-handed neutrino states. The term of interest is

$$\mathcal{L}_{Yukawa,\Delta} = \sum_{\alpha,\beta=e,\mu,\tau} (Y^\nu_{\alpha\beta} L^T_{\alpha L} C^\dagger i\tau^2 \Delta L_{\beta L} + h.c) \tag{3.12}$$

which can be written in terms of the triplet components as

$$\begin{aligned} \mathcal{L}_{Yukawa,\Delta} =& \sum_{\alpha,\beta=e,\mu,\tau} Y_{\alpha\beta}\nu^T_{\alpha L}C^\dagger\delta^0\nu_{\beta L} - \frac{Y_{\alpha\beta}}{\sqrt{2}}\left(\nu^T_{\alpha L}C^\dagger\delta^+\ell_{\beta L} + \ell^T_{\alpha L}C^\dagger\delta^+\nu_{\beta L}\right) \\ &- Y_{\alpha\beta}\ell^T_{\alpha L}C^\dagger\delta^{++}\ell_{\beta L} + h.c \end{aligned} \tag{3.13}$$

The potential given by eqn. (3.11) exhausts all possible renormalizable, gauge-invariant operators. For. eg. a term of the type $\lambda_5\Phi^\dagger\Delta^\dagger\Delta\Phi$ is legitimate, but it can actually be projected on the operators with coefficients λ_1 and λ_4 by means of the identity $\Phi^\dagger\Delta^\dagger\Delta\Phi + \Phi^\dagger\Delta\Delta^\dagger\Phi = \Phi^\dagger\Phi Tr(\Delta^\dagger\Delta)$. This is valid because Δ is a traceless 2×2 matrix. We need only to redefine λ_1 and λ_4 as $\lambda_1 + \lambda_5 \to \lambda_1$, $\lambda_4 - \lambda_5 \to \lambda_4$. Thus, the potential depends on five independent dimensionless couplings λ and $\lambda_i, (i = 1, ..., 4)$ and three mass parameters m_H^2, M_Δ^2 and μ. Throughout the discussion to follow, these parameters are assumed to be real. V depends on five complex or equivalently ten real scalar fields.

Assuming that spontaneous electroweak symmetry breaking takes place at some electrically neutral point in the field space and denoting the corresponding VEV's by

$$\langle\Delta\rangle = \begin{pmatrix} 0 & 0 \\ v_t/\sqrt{2} & 0 \end{pmatrix} \qquad \text{and} \qquad \langle\Phi\rangle = \begin{pmatrix} 0 \\ v_d/\sqrt{2} \end{pmatrix} \tag{3.14}$$

the following conditions are obtained after minimization of the potential

$$M_{\Delta}^{2} = \frac{2\mu v_d^2 - \sqrt{2}(\lambda_1 + \lambda_4)v_d^2 v_t - 2\sqrt{2}(\lambda_2 + \lambda_3)v_t^3}{2\sqrt{2}v_t} \tag{3.15}$$

$$m_H^2 = \frac{\lambda v_d^2}{4} - \sqrt{2}\mu v_t + \frac{(\lambda_1 + \lambda_4)}{2} v_t^2 \tag{3.16}$$

Since we are interested in a heavy Higgs triplet, $M_{\Delta}^{2} > v_d^2/2$ or $v_t \ll v_d$, so all terms proportional to λ_1, λ_2, λ_3 and λ_4 are neglected.

Thus,

$$M_{\Delta}^{2} = \frac{\mu v_d^2}{\sqrt{2}v_t} \tag{3.17}$$

After symmetry breaking, the term given by eqn. (3.12) becomes

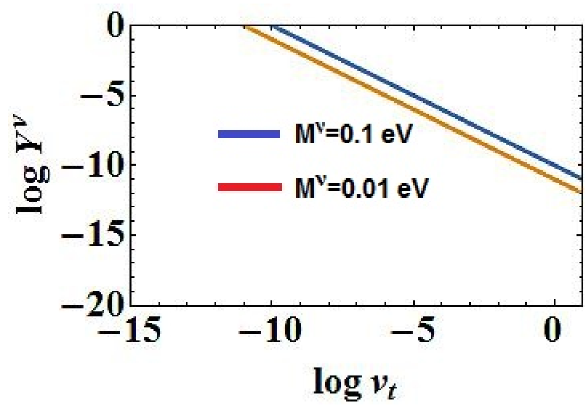
$$\begin{aligned} \mathcal{L}_{Yukawa,\Delta} &= \sum_{\alpha,\beta=e,\mu,\tau} Y_{\alpha\beta}^{\nu} \begin{pmatrix} \nu_{\alpha L}^{T} & \ell_{\alpha L}^{T} \end{pmatrix} C^{\dagger} \begin{pmatrix} v_t/\sqrt{2} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \nu_{\beta L} \\ \ell_{\beta L} \end{pmatrix} + h.c \\ &= \sum_{\alpha,\beta=e,\mu,\tau} Y_{\alpha\beta}^{\nu} \nu_{\alpha L}^{T} \frac{v_t}{\sqrt{2}} C^{\dagger} \nu_{\beta L} + h.c \\ &= \frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} M_{\alpha\beta}^{\nu} \nu_{\alpha L}^{T} C^{\dagger} \nu_{\beta L} + h.c \end{aligned} \tag{3.18}$$

where the Majorana neutrino mass-matrix is given by

$$M_{\alpha\beta}^{\nu} = \sqrt{2} Y_{\alpha\beta}^{\nu} v_t \tag{3.19}$$

The current upper bounds on neutrino masses ($\sim 10^{-10}$ GeV) obtained from cosmological and oscillation data severely constrains the values of $\boldsymbol{Y}^{\nu}$ and v_t. A graph depicting this is shown below for $\boldsymbol{M}^{\nu}$=0.1 eV and $\boldsymbol{M}^{\nu}$=0.01 eV.

Figure 1 Variation of $\boldsymbol{Y}^{\nu}$ with v_t for fixed neutrino mass (logarithmic scale)



Using eqn. (3.17), an alternate expression for the neutrino masses can be obtained as

$$M^{\nu}_{\alpha\beta} = Y^{\nu}_{\alpha\beta}\frac{\mu v_d^2}{M_\Delta^2} \tag{3.20}$$

This is the key relation for the type-II scenario. The smallness of the neutrino masses can be explained by taking heavy Higgs triplets such that M_Δ is large.

Table 2: Order of magnitudes of $\boldsymbol{Y}^\nu$, v_t and μ for different values of $\boldsymbol{M}^\nu$ and M_Δ.

$\boldsymbol{M}^\nu$(eV)	$\boldsymbol{Y}^\nu$(GeV)	v_t(GeV)	μ (for $M_\Delta \sim v_d$=246 GeV)	μ (for $M_\Delta \sim 10^{16}$ GeV)
$\sim$0.1	$\sim 10^{-5}$	$\sim 10^{-5}$	$\sim 10^{-5}$	$\sim 10^{23}$
$\sim$0.1	$\mathcal{O}(1)$	$\sim 10^{-10}$	$\sim 10^{-10}$	$\sim 10^{18}$
$\sim$0.1	$\sim 10^{-11}$	$\sim$4.6	$\sim$4.6	$\sim 10^{28}$
$\sim$0.01	$\sim 10^{-5}$	$\sim 10^{-6}$	$\sim 10^{-6}$	$\sim 10^{22}$
$\sim$0.01	$\mathcal{O}(1)$	$\sim 10^{-11}$	$\sim 10^{-11}$	$\sim 10^{19}$
$\sim$0.01	$\sim 10^{-12}$	$\sim$ 4.6	$\sim$ 4.6	$\sim 10^{28}$

From the table, it is visible that for large values of the Yukawa couplings $\mathcal{O}(1)$, v_t is of the order of neutrino masses. On the other hand, if $\boldsymbol{Y}^\nu$'s are of the order of electron couplings, then also $v_t = 10^{-2}$ MeV. Even with v_t=4.6 Gev or the upper bound set by electroweak precision measurements (see section 3.4), $\boldsymbol{Y}^\nu \sim 10^{-11}$. Thus, the Yukawa couplings have to be very small to account for small neutrino masses. However, there are six such matrix elements ($\boldsymbol{Y}^\nu$ is symmetric) and hence such an adjustment makes the scenario more fine-tuned than the one in which we can tune a single quantity v_t. If we consider light Higgs triplets ($M_\Delta \sim v_d$), which are within the reach of LHC, then μ is of the order of v_t. On the other hand, for heavy Higgs triplets, μ which has the dimensions of energy is quite large.

3.3 Charged Lepton couplings with the triplet

The charged lepton couplings with the triplet are given by

$$\sum_{\alpha,\beta} -\frac{Y_{\alpha\beta}}{\sqrt{2}}\left(\nu^T_{\alpha L}C^\dagger\delta^+\ell_{\beta L} + \ell^T_{\alpha L}C^\dagger\delta^+\nu_{\beta L}\right) + h.c \tag{3.21}$$

and

$$\sum_{\alpha,\beta} Y_{\alpha\beta}\ell_{\alpha L}^{T}C^{\dagger}\delta^{++}\ell_{\beta L} + h.c \tag{3.22}$$

The second coupling is of particular interest. In the physical particle spectrum, the doubly charged component δ^{++} is also a mass eigenstate. Representing this mass eigenstate by H^{++}, we can clearly see that decays of the doubly charged scalar to same-sign dileptons are possible

$$H^{\pm\pm} \to \ell_{\alpha}^{\pm}\ell_{\beta}^{\pm} \qquad (\alpha, \beta = e, \mu, \tau) \tag{3.23}$$

The decay width is given by

$$\Gamma(H^{\pm\pm} \to \ell_{\alpha}^{\pm}\ell_{\beta}^{\pm}) = \frac{S m_{H^{\pm\pm}} \mid Y_{\alpha\beta} \mid^2}{8\pi} \tag{3.24}$$

where

$$\begin{aligned} S &= 1, \qquad \alpha = \beta \\ &= 2, \qquad \alpha \neq \beta \end{aligned}$$

So, it is governed by the Yukawa coupling matrix-elements $Y_{\alpha\beta}$ which also appear in the Majorana neutrino mass matrix via eqn. (3.19). Hence, experimental signatures of decays into same-sign dileptons can serve as a probe of the Yukawa coupling structure. Moreover, if $m_{H^{\pm\pm}}$ lies in the TeV region (within the reach of the LHC) and v_t is sufficiently small (< 0.1 MeV), then the dominant decay mode of $H^{\pm\pm}$ is into like-sign dileptons. Assuming that type II see-saw is the dominant contributor to neutrino mass and only the above mentioned decays are taken into account, the branching ratio can be written as

$$\begin{aligned} BR(H^{\pm\pm} \to \ell_{\alpha}^{\pm}\ell_{\beta}^{\pm}) &= \frac{S \mid Y_{\alpha\beta} \mid^2}{\sum \mid Y_{\alpha\beta} \mid^2} \\ &= \frac{S\left[V_{PMNS}^{*} diag(m_1, m_2 e^{i\varphi_1}, m_3 e^{i\varphi_2}) V_{PMNS}^{\dagger}\right]_{\alpha\beta}^{2}}{\sum_k m_k^2} \end{aligned} \tag{3.25}$$

where the neutrino mass matrix $\boldsymbol{M}$ is diagonalized as

$$M_{\alpha\beta} = \left[V_{PMNS}^{*} diag(m_1, m_2 e^{i\varphi_1}, m_3 e^{i\varphi_2}) V_{PMNS}^{\dagger}\right]_{\alpha\beta} \tag{3.26}$$

where V_{PMNS} is the Pontecorvo-Maki-Nakagawa-Sakata neutrino mixing matrix and φ_1, φ_2 are Majorana phases. From the above expression, it is evident that the BR is dependent only on neutrino parameters. Thus, doubly charged Higgs decays can prove useful in getting an idea of the absolute masses of neutrinos as well as Majorana phases which are difficult to obtain through neutrino oscillation data [12-15].

The coupling to the singly-charged scalar field given by eqn. (3.21) likewise shows that one has the possibility of decays

$$H^{\pm} \to \ell^{\pm}_{\alpha} \nu_{\beta} \qquad (\alpha, \beta = e, \mu, \tau) \tag{3.27}$$

The coupling being proportional to the Yukawa matrix elements, there is again the scope for probing the Yukawa flavor structure through experimental observation of such decays.

3.4 Constraints from electroweak precision measurements

In this extension of the Standard Model, the Z and W gauge boson masses as calculated from the kinetic part of the Lagrangian, (given by eqn. (3.8)) $\mathcal{L}_{kinetic} = (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) + Tr(\mathcal{D}_\mu \Delta)^\dagger (\mathcal{D}^\mu \Delta)$, are

$$M_Z^2 = \frac{g^2(v_d^2 + 4v_t^2)}{4\cos^2\theta_W} \tag{3.28}$$

$$M_W^2 = \frac{g^2(v_d^2 + 2v_t^2)}{4} \tag{3.29}$$

In the Standard Model, the custodial symmetry ensures that the ρ parameter given by $\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W}$ is equal to unity at tree level. However, with the added Higgs triplet, the modified ρ parameter is given by

$$\rho = \frac{v_d^2 + 2v_t^2}{v_d^2 + 4v_t^2} \neq 1 \tag{3.30}$$

and actually $\rho < 1$ at the tree-level. Since we are interested in the limit $v_t \ll v_d$ we can rewrite ρ as

$$\rho \simeq 1 - 2\frac{v_t^2}{v_d^2} = 1 + \delta\rho \tag{3.31}$$

where $\delta\rho = -2\frac{v_t^2}{v_d^2} < 0$. Also, for the correct electroweak scale, $v = \sqrt{v_d^2 + 2v_t^2} = 246$ GeV. This puts a further constraint on v_t. Thus, the model with extended scalar sector will remain viable as far as the experimentally driven values of $\delta\rho$ are compatible with a negative number.

Using the experimental value of the ρ-parameter, we can put an upper bound on the triplet VEV. After subtracting the Standard Model contributions to the ρ-parameter, the value of ρ obtained is $\rho_0 = 1.0008^{+0.0017}_{-0.0007}$. This is obtained from a global fit including the direct search limits on the standard Higgs boson. The value seems incompatible with

$\delta\rho < 0$ ($\delta\rho = 0.0001 > 0$) and thus seems to exclude the Higgs Triplet Model. However, at the 2σ level, one obtains $\rho_0 = 1.0004^{+0.0029}_{-0.0011}$ [16] which satisfies $\delta\rho < 0$. Relaxing the direct Higgs limit leads to $\rho_0 = 1.0008^{+0.0017}_{-0.0010}$ again consistent with $\delta\rho < 0$.

From the last two ρ_0 values stated above, the values of $x = v_t/v_d$ obtained from eqns. (3.30) or (3.31) are ~ 0.0187 and ~ 0.01 respectively giving the upper bounds on v_t to be of the range 2.5-4.6 GeV.

Chapter 4

Type III see-saw mechanism

From the last two chapters, it is evident that an augmentation of the usual Standard Model particle content is essential to explain the observed smallness of neutrino masses. In this chapter, the third type of realization of the see-saw mechanism is discussed, which has some similarities with the type I see-saw. Here also, the fermionic sector is expanded but by adding $SU(2)_L$ fermion triplets with $Y = 0$. The resulting mass matrix has a form similar to the one obtained in the type I see-saw. However, this model has a distinct advantage that the $SU(2)_L$ triplets have gauge interactions and hence comparatively larger cross-sections, which offer test possibilities in a more comprehensive manner upto the TeV range. The testability of all the three kinds of see-saw mechanisms form the subject matter of Chapter 5.

4.1 The model

In the type III scenario [17], the Standard Model is enlarged by adding atleast two fermionic triplets, $\mathbf{\Sigma}_j$ which transform under $SU(2)_L \otimes U(1)_Y$ as: $(3, 0)$. Each triplet is composed of three Weyl spinors which are taken to be left-handed in the present discussion. In terms of Cartesian components, $\mathbf{\Sigma}_j = (\Sigma^1_{jL}, \Sigma^2_{jL}, \Sigma^3_{jL})$ and the 2×2 matrix representation of $\mathbf{\Sigma}_j$ is

$$\Sigma_{jL} = \frac{1}{\sqrt{2}}\mathbf{\Sigma}_j.\boldsymbol{\tau} = \frac{1}{\sqrt{2}}\begin{pmatrix} \sum^3_{jL} & \sum^1_{jL} - i\sum^2_{jL} \\ \sum^1_{jL} + i\sum^2_{jL} & -\sum^3_{jL} \end{pmatrix} \tag{4.1}$$

which transforms under $SU(2)_L \otimes U(1)_Y$ as $\sum_{jL} = \mathcal{U}\sum_{jL}\mathcal{U}^\dagger$, where $\mathcal{U} = e^{i\theta^a(x)I^a}$, as defined in section 3.2. The electric charges can again be determined using the Gell-Mann-Nishijima relation, which in this picture becomes $Q = I_3$. Denoting the electric charge eigenstates by Σ_j^0, Σ_j^+ and Σ_j^3, we can write the matrix representation of the triplet as

$$\Sigma_{jL} = \begin{pmatrix} \Sigma_{jL}^0/\sqrt{2} & \Sigma_{jL}^+ \\ \Sigma_{jL}^- & -\Sigma_{jL}^0/\sqrt{2} \end{pmatrix} \tag{4.2}$$

where

$$\Sigma_{jL}^0 = \Sigma_{jL}^3, \quad \Sigma_{jL}^+ = \frac{1}{\sqrt{2}}(\Sigma_{jL}^1 - i\Sigma_{jL}^2), \quad \Sigma_{jL}^- = \frac{1}{\sqrt{2}}(\Sigma_{jL}^1 + i\Sigma_{jL}^2) \tag{4.3}$$

From here onwards, I will drop the generation index 'j' in the expressions for the triplet. Using the property that

$$\begin{aligned} \Sigma_L^{+C} &= \frac{\Sigma_L^{1C} - i\Sigma_L^{2C}}{\sqrt{2}} \\ &= \frac{-\gamma^0 C(\Sigma_L^{1*} - i\Sigma_L^{2*})}{\sqrt{2}} \\ &= \frac{-\gamma^0 C(\Sigma_L^1 + i\Sigma_L^2)^*}{\sqrt{2}} \\ &= -\gamma^0 C\Sigma_L^{-*} \end{aligned} \tag{4.4}$$

and likewise $\Sigma_L^{-C} = -\gamma^0 C\Sigma_L^{+*}$, the charge conjugate of the triplet can be written as

$$\begin{aligned} \Sigma_L^C &= \begin{pmatrix} -\gamma^0 C\Sigma_L^{0*}/\sqrt{2} & -\gamma^0 C\Sigma_L^{+*} \\ -\gamma^0 C\Sigma_L^{-*} & \gamma^0 C\Sigma_L^{0*}/\sqrt{2} \end{pmatrix} \\ &= \begin{pmatrix} \Sigma_L^{0C}/\sqrt{2} & \Sigma_L^{-C} \\ \Sigma_L^{+C} & -\Sigma_L^{0C}/\sqrt{2} \end{pmatrix} \end{aligned} \tag{4.5}$$

This charge conjugate Σ_L^C represents right-handed fields which can be easily checked by operating $L = (1-\gamma^5)/2$ and using the following properties of the chirality projection operators to see that it indeed vanishes.

$$L\gamma^0 = \gamma^0 R, \quad RC = CR^T \quad \text{and} \quad RL = 0 \tag{4.6}$$

The following two forms of the triplet are also written for convenience in writing the interaction Lagrangian which will be discussed in the coming section.

$$\overline{\Sigma}_L = \begin{pmatrix} \overline{\Sigma}_L^0/\sqrt{2} & (\overline{\Sigma}_L)^- \\ (\overline{\Sigma}_L)^+ & \overline{\Sigma}_L^0/\sqrt{2} \end{pmatrix}, \qquad \overline{\Sigma_L^C} = \begin{pmatrix} \overline{\Sigma_L^{0C}}/\sqrt{2} & \overline{\Sigma_L^{+C}} \\ \overline{\Sigma_L^{-C}} & -\overline{\Sigma_L^{0C}}/\sqrt{2} \end{pmatrix} \tag{4.7}$$

4.1.1 The Lagrangian

The renormalizable Lagrangian is given by [18]

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{kinetic} + \mathcal{L}_M + \mathcal{L}_Y \tag{4.8}$$

where $\mathcal{L}_{SM}$ is the Standard Model Lagrangian and

$$\begin{aligned} \mathcal{L}_{kinetic} &= Tr[\overline{\Sigma}_L i \not{D} \Sigma_L] \\ \mathcal{L}_M &= -\frac{1}{2} Tr[\overline{\Sigma_L^C} M_\Sigma \Sigma_L + \overline{\Sigma_L} M_\Sigma^* \Sigma_L^C] \\ \mathcal{L}_Y &= -(\overline{L}_L \sqrt{2} Y_\Sigma^\dagger \Sigma_L^C \tilde{\Phi} + \tilde{\Phi}^\dagger \overline{\Sigma_L^C} \sqrt{2} Y_\Sigma L_L) \end{aligned} \tag{4.9}$$

The term in the first line represents the kinetic term while that in the second line gives the mass term for the triplet. The Yukawa interaction of the triplet with the modified Higgs doublet, given by eqn. (1.14), is represented by the terms in the last line.

The covariant derivative is given by

$$\mathcal{D}_\mu \Sigma_L = \partial_\mu \Sigma_L + ig[I^a A_\mu^a, \Sigma_L] \tag{4.10}$$

The term containing B_μ is absent because $Y = 0$ for the triplet Σ_L. Now, I will write the terms in the Lagrangian relevant for the charged lepton and neutrino mass matrices in terms of the triplet components.

$$\begin{aligned} \mathcal{L}_M &= -\frac{1}{2} Tr[\overline{\Sigma_L^C} M_\Sigma \Sigma_L + \overline{\Sigma_L} M_\Sigma^* \Sigma_L^C] \\ &= -\frac{1}{2}[(\overline{\Sigma_L^{0C}} M_\Sigma \Sigma_L^0 + \overline{\Sigma_L^{+C}} M_\Sigma \Sigma_L^- + \overline{\Sigma_L^{-C}} M_\Sigma \Sigma_L^+) \\ &+ (\overline{\Sigma_L^0} M_\Sigma^* \Sigma_L^{0C} + \overline{\Sigma_L^-} M_\Sigma^* \Sigma_L^{+C} + \overline{\Sigma_L^+} M_\Sigma^* \Sigma_L^{-C})] \end{aligned} \tag{4.11}$$

The Yukawa interaction can be written as

$$\begin{aligned} \mathcal{L}_Y &= -(\overline{L}_L \sqrt{2} Y_\Sigma^\dagger \Sigma_L^C \tilde{\Phi} + \tilde{\Phi}^\dagger \overline{\Sigma_L^C} \sqrt{2} Y_\Sigma L_L) \\ &= -[(\overline{\boldsymbol{\nu'}}_L Y_\Sigma^\dagger \Sigma_L^{0C} \phi^{0*} - \sqrt{2}\overline{\boldsymbol{\nu'}}_L Y_\Sigma^\dagger \Sigma_L^{-C} \phi^- + \sqrt{2}\overline{\boldsymbol{\ell}'}_L Y_\Sigma^\dagger \Sigma_L^{+C} \phi^{0*} + \overline{\boldsymbol{\ell}'}_L Y_\Sigma^\dagger \Sigma_L^{0C} \phi^-) \\ &+ (\phi^0 \overline{\Sigma^{0C}} Y_\Sigma \boldsymbol{\nu'}_L - \sqrt{2}\phi^+ \overline{\Sigma_L^{-C}} Y_\Sigma \boldsymbol{\nu'}_L + \sqrt{2}\phi^0 \overline{\Sigma_L^{+C}} Y_\Sigma \boldsymbol{\ell}'_L + \phi^+ \overline{\Sigma_L^{0C}} Y_\Sigma \boldsymbol{\ell}'_L)] \end{aligned} \tag{4.12}$$

The physical particles are charged Dirac fermions E' and neutral Majorana fermions N' (primes refer to weak interaction eigenstates throughout our discussion).

Define $E' = \Sigma_L^- + \Sigma_L^{+C}$ and $N' = \Sigma_L^0 + \Sigma_L^{0C}$. With our choice of left-handed triplets, the following relations are easy to obtain

$$E'_L = \Sigma_L^-, \quad E'_R = \Sigma_L^{+C} \tag{4.13}$$

$$N'_L = \Sigma_L^0, \quad N'_R = \Sigma_L^{0C} \tag{4.14}$$

From $E'_R = \Sigma_L^{+C}$ and using the relation $C^{-1}\gamma^\mu C = -\gamma^T$ for the charge conjugation operator, we can write $\Sigma_L^+ = E_R'^C$. A bunch of similar relations can now be easily written which will come very handy when we express the expressions obtained in eqns. (4.13) and (4.14) in terms of the physical fields.

For the charged Dirac fields E'

$$\begin{aligned} E_L'^C &= \Sigma_L^{-C}, \quad E_R'^C = \Sigma_L^+ \\ \overline{E'}_L &= \overline{\Sigma_L^-}, \quad \overline{E'}_R = \overline{\Sigma_L^{+C}} \\ \overline{E_L'^C} &= \overline{\Sigma_L^{-C}}, \quad \overline{E_R'^C} = \overline{\Sigma_L^+} \end{aligned} \tag{4.15}$$

while for the neutral Majorana fields N'

$$\begin{aligned} N_L'^C &= \Sigma_L^0, \quad N_R'^C = \Sigma_L^{0C} \\ \overline{N'}_L &= \overline{\Sigma_L^0}, \quad \overline{N'}_R = \overline{\Sigma_L^{0C}} \\ \overline{N_L'^C} &= \overline{\Sigma_L^0}, \quad \overline{N_R'^C} = \overline{\Sigma_L^{0C}} \end{aligned} \tag{4.16}$$

Eqns. (4.11) and (4.12) in terms of physical fields can be written as

$$\begin{aligned} \mathcal{L}_M = &-\frac{1}{2}[(\overline{N'}_R M_\Sigma N'_L + \overline{E'}_R M_\Sigma E'_L + \overline{E_L'^C} M_\Sigma E_R'^C) \\ &+ (\overline{N'}_L M_\Sigma^* N'_R + \overline{E'}_L M_\Sigma^* E'_R + \overline{E_R'^C} M_\Sigma^* E_L'^C)] \end{aligned} \tag{4.17}$$

Using $\overline{E_L'^C} M_\Sigma E_R'^C = \overline{E'_R} M_\Sigma E'_L$ and $\overline{E_R'^C} M_\Sigma^* E_L'^C = \overline{E'_L} M_\Sigma^* E'_R$, the above equation can be re-written as

$$\mathcal{L}_M = -\frac{1}{2}(\overline{N'_R} M_\Sigma N'_L + \overline{N'_L} M_\Sigma^* N'_R) - (\overline{E'_R} M_\Sigma E'_L + \overline{E'_L} M_\Sigma^* E'_R) \tag{4.18}$$

After spontaneous symmetry breaking,

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

so that eqn. (4.12) in terms of physical fields becomes

$$\mathcal{L}_Y = -\left[\frac{v}{\sqrt{2}}\overline{\boldsymbol{\nu}'_L}Y_\Sigma^\dagger N'_R + v\overline{\boldsymbol{\ell}'_L}Y_\Sigma^\dagger E'_R + \frac{v}{\sqrt{2}}\overline{N'_R}Y_\Sigma\boldsymbol{\nu}'_L + v\overline{E'_R}Y_\Sigma\boldsymbol{\ell}'_L\right] \tag{4.19}$$

Using the Majorana nature of $\boldsymbol{\nu}'$ and N' and the properties of the charge conjugation operator C, we can write $\overline{\boldsymbol{\nu}'_L}Y_\Sigma^\dagger N'_R = \overline{N'^C_R}(Y_\Sigma^T)^\dagger\boldsymbol{\nu}'^C_L$ and $\overline{N'_L}(Y_\Sigma^T)^\dagger\boldsymbol{\nu}'^C_L = \overline{N'_R}Y_\Sigma\boldsymbol{\nu}'_L$ so that the above equation can be re-written as

$$\mathcal{L}_Y = -v\left[\overline{\boldsymbol{\ell}'_L}Y_\Sigma^\dagger E'_R + \overline{E'_R}Y_\Sigma\boldsymbol{\ell}'_L\right] - \frac{v}{2\sqrt{2}}\left[\overline{\boldsymbol{\nu}'_L}Y_\Sigma^\dagger N'_R + \overline{N'^C_R}(Y_\Sigma^T)^\dagger\boldsymbol{\nu}'^C_L + \overline{N'_R}Y_\Sigma\boldsymbol{\nu}'_L + \overline{N'_L}(Y_\Sigma^T)^\dagger\boldsymbol{\nu}'^C_L\right] \tag{4.20}$$

Combining results from eqns. (4.18) and (4.19) with the Standard Model charged lepton-Higgs Yukawa interaction

$$\mathcal{L}^\ell_{SM,Yukawa} = -(\overline{\boldsymbol{\ell}'_L}Y_\ell^\dagger\frac{v}{\sqrt{2}}\boldsymbol{\ell}'_R + \overline{\boldsymbol{\ell}'_R}Y_\ell\frac{v}{\sqrt{2}}\boldsymbol{\ell}'_L) \tag{4.21}$$

the terms can be arranged as charged leptonic and neutral leptonic (neutrino) mass matrices

$$\begin{aligned}\mathcal{L}_{mass} &= \mathcal{L}_M + \mathcal{L}_Y + \mathcal{L}_{SM,Yukawa}\\ &= \mathcal{L}_{CLepton} + \mathcal{L}_{NLepton}\end{aligned} \tag{4.22}$$

where

$$\mathcal{L}_{CLepton} = -\left[\begin{pmatrix}\overline{\boldsymbol{\ell}'_R} & \overline{E'_R}\end{pmatrix}\begin{pmatrix}m_\ell & 0\\ Y_\Sigma v & M_\Sigma\end{pmatrix}\begin{pmatrix}\boldsymbol{\ell}'_L\\ E'_L\end{pmatrix} + \begin{pmatrix}\overline{\boldsymbol{\ell}'_L} & \overline{E'_L}\end{pmatrix}\begin{pmatrix}m_\ell^* & Y_\Sigma^\dagger v\\ 0 & M_\Sigma^*\end{pmatrix}\begin{pmatrix}\boldsymbol{\ell}'_R\\ E'_R\end{pmatrix}\right] \tag{4.23}$$

and

$$\begin{aligned}\mathcal{L}_{NLepton} = -\frac{1}{2}\Big[&\begin{pmatrix}\overline{\boldsymbol{\nu}'^C_L} & \overline{N'_R}\end{pmatrix}\begin{pmatrix}0 & \frac{Y_\Sigma^T v}{\sqrt{2}}\\ \frac{Y_\Sigma v}{\sqrt{2}} & M_\Sigma\end{pmatrix}\begin{pmatrix}\boldsymbol{\nu}'_L\\ N'_L\end{pmatrix}\\ &+\begin{pmatrix}\overline{\boldsymbol{\nu}'_L} & \overline{N'_L}\end{pmatrix}\begin{pmatrix}0 & \frac{Y_\Sigma^\dagger v}{\sqrt{2}}\\ \frac{Y_\Sigma^* v}{\sqrt{2}} & M_\Sigma^*\end{pmatrix}\begin{pmatrix}\boldsymbol{\nu}'^C_L\\ N'_R\end{pmatrix}\Big]\end{aligned} \tag{4.24}$$

Here $\boldsymbol{\ell}'_L$ and E'_L are 3×1 column matrices and hence the mass matrices in eqns. (4.23) and (4.24) are 6×6 matrices. Eqn. (4.23) shows that there is mixing between doublet and triplet charged leptons in the type III scenario unlike the other two types of see-saw. This mixing results in FCNC's involving charged leptons at the tree level. Also, as we will see in the next section, triplet leptons have gauge interactions which leads to different phenomenology.

4.2 Diagonalization of the mass matrices

For detailed investigations of the phenomenology of the type III see-saw mechanism, one needs to understand the gauge interactions in the mass basis. This is achieved by studying the mass matrices better and their diagonalization. The diagonalization is achieved using unitary matrices $U^{L,R}$ for charged lepton case and U^0 for the neutrino case such that

$$U^{R\dagger}\begin{pmatrix} m_\ell & 0 \\ Y_\Sigma v & M_\Sigma \end{pmatrix} U^L = \begin{pmatrix} m^\ell_{light} & 0 \\ 0 & m^\ell_{heavy} \end{pmatrix} \tag{4.25}$$

and

$$U^{0^T}\begin{pmatrix} 0 & \frac{Y_\Sigma^T v}{\sqrt{2}} \\ \frac{Y_\Sigma v}{\sqrt{2}} & M_\Sigma \end{pmatrix} U^0 = \begin{pmatrix} m^\nu_{light} & 0 \\ 0 & m^\nu_{heavy} \end{pmatrix} \tag{4.26}$$

where

$$U^{L,R} = \begin{pmatrix} U^{L,R}_{\ell\ell} & U^{L,R}_{\ell E} \\ U^{L,R}_{E\ell} & U^{L,R}_{EE} \end{pmatrix} \text{and} \qquad U^0 = \begin{pmatrix} U^0_{\nu\nu} & U^0_{\nu N} \\ U^0_{N\nu} & U^0_{NN} \end{pmatrix} \tag{4.27}$$

The weak and mass eigenstates are related as

$$\begin{pmatrix} \boldsymbol{\ell}'_{L,R} \\ E'_{L,R} \end{pmatrix} = U^{L,R}\begin{pmatrix} \boldsymbol{\ell}_{L,R} \\ E_{L,R} \end{pmatrix}, \qquad \begin{pmatrix} \boldsymbol{\nu}'_L \\ N'_L \end{pmatrix} = U^0 \begin{pmatrix} \boldsymbol{\nu}_L \\ N_L \end{pmatrix} \tag{4.28}$$

It should be noted that the individual 3×3 block matrices in the diagonalizing matrices defined above are not unitary. In principle, the matrices $U_{L,R}$ and U^0 can be expressed in terms of Y_Σ, m_ℓ and M_Σ. For the see-saw mechanism to work, $Y_\Sigma v M_\Sigma^{-1}$ should be small and hence one can expand $U^{L,R}$ and U^0 in powers of $Y_\Sigma v M_\Sigma^{-1}$ keeping track of the leading order contributions. Since M_Σ is quite large, so it is reasonable to keep expressions up to $\mathcal{O}(Y_\Sigma^2 v^2 M_\Sigma^{-2})$ in the basis where m_ℓ and M_Σ are already diagonalized. Under this approximation, the following expressions [19] for the matrices are obtained (worked out in Appendix B)

$$\begin{aligned}
&U^L_{\ell\ell} = 1-\epsilon, \quad U^L_{\ell E} = Y_\Sigma^\dagger M_\Sigma^{-1} v, \quad U^L_{E\ell} = -M_\Sigma^{-1} Y_\Sigma v, \quad U^L_{EE} = 1-\epsilon', \\
&U^R_{\ell\ell} = 1, \quad U^R_{\ell E} = m_\ell Y_\Sigma^\dagger M_\Sigma^{-2} v, \quad U^R_{E\ell} = -M_\Sigma^{-2} Y_\Sigma m_\ell v, \quad U^R_{EE} = 1, \\
&U^0_{\nu\nu} = (1-\epsilon/2)V_{PMNS}, \quad U^0_{\nu N} = Y_\Sigma^\dagger M_\Sigma^{-1} v/\sqrt{2}, \quad U^0_{N\nu} = -M_\Sigma^{-1} Y_\Sigma U^0_{\nu\nu} v/\sqrt{2}, \\
&U^0_{NN} = 1-\epsilon'/2, \qquad \epsilon = Y_\Sigma^\dagger M_\Sigma^{-2} Y_\Sigma v^2/2, \quad \epsilon' = M_\Sigma^{-1} Y_\Sigma Y_\Sigma^\dagger M_\Sigma^{-1} v^2/2
\end{aligned} \tag{4.29}$$

The light-neutrino mass matrix is given by

$$m^\nu_{light} \simeq -\frac{v^2}{2} Y_\Sigma^T M_\Sigma^{-1} Y_\Sigma \tag{4.30}$$

whereas the heavy triplet neutrinos have masses of the order M_Σ. This expression is similar to the one obtained in eqn. (2.45) for type I see-saw mechanism with $Y_\Sigma v$ playing the role of M^D and M_Σ mimicking the heavy M^R of the type I picture. If we take in general N heavy fermion triplets Σ_{jL}, then in the final spectrum there are 3 light neutrinos and N heavy neutrinos. This can be checked by looking at the dimensions of the matrices constituting the light and heavy neutrino mass matrices respectively.

In the general formalism, M_Σ is an $N \times N$ complex, symmetric matrix and Y_Σ is an $N \times 3$ complex matrix, so that

$$m_\nu^{light} = -\frac{v^2}{2} Y_\Sigma^T M_\Sigma^{-1} Y_\Sigma, \qquad m_\nu^{heavy} \simeq M_\Sigma \tag{4.31}$$

are 3×3 and $N \times N$ matrices respectively.

Similar to the discussion that follows eqn. (2.45) in chapter 2, it can once again be shown that for addition of only one fermion triplet, there is only one massive light neutrino which is insufficient to account for the two kinds of splittings Δm^2_{21} and $\mid \Delta m^2_{31} \mid$ observed in neutrino oscillations.

For the case of two added heavy fermion triplets, there are two non-zero eigenvalues for the light neutrino mass matrix and thus atleast two heavy $SU(2)_L$ fermion triplets are sufficient to explain the empirical data.

From eqn. (4.30), it is once again evident that for $Y_\Sigma \sim \mathcal{O}(1)$, the order of the triplet mass, M_Σ should be $10^{14} - 10^{16}$ GeV or the GUT scale. However, if the Yukawa couplings are small, of the order of electron Yukawa couplings or smaller, then M_Σ can be of the order of TeV and hence accessible to the LHC.

4.3 Gauge Interactions

The kinetic term is given by eqn. (4.9). In terms of triplet components, it can be written as

$$\begin{aligned}\mathcal{L}_{kinetic} = Tr(\overline{\Sigma}_L i\not{D}\Sigma_L) = (i\overline{\Sigma^0_L}\not{\partial}\Sigma^0_L + i\overline{\Sigma^+_L}\not{\partial}\Sigma^+_L + i\overline{\Sigma^-_L}\not{\partial}\Sigma^-_L) \\ - g(\overline{\Sigma^0_L}\gamma^\mu\Sigma^-_L W^+_\mu + \overline{\Sigma^-_L}\gamma^\mu\Sigma^0_L W^-_\mu) + g(\overline{\Sigma^0_L}\gamma^\mu\Sigma^+_L W^-_\mu + \overline{\Sigma^+_L}\gamma^\mu\Sigma^0_L W^+_\mu) \\ + e(\overline{\Sigma^-_L}\gamma^\mu\Sigma^-_L A_\mu - \overline{\Sigma^+_L}\gamma^\mu\Sigma^+_L A_\mu) + g\cos\theta_W(\overline{\Sigma^-_L}\gamma^\mu\Sigma^-_L Z_\mu - \overline{\Sigma^+_L}\gamma^\mu\Sigma^+_L Z_\mu)\end{aligned} \tag{4.32}$$

In terms of weak interaction eigenstates,

$$\begin{aligned}\mathcal{L}_{kinetic} &= (i\overline{N'_L}\not{\partial}N'_L + i\overline{E'^C_R}\not{\partial}E'^C_R + i\overline{E'_L}\not{\partial}E'_L) \\ &- g(\overline{N'_L}\gamma^\mu E'_L W^+_\mu + \overline{E'_L}\gamma^\mu N'_L W^-_\gamma) + g(\overline{N'_L}\gamma^\mu E'^C_R W^-_\mu + \overline{E'^C_R}\gamma^\mu N'_L W^+_\mu) \\ &+ e(\overline{E'_L}\gamma^\mu E'_L A_\mu - \overline{E'^C_R}\gamma^\mu E'^C_R A_\mu) + g\cos\theta_W(\overline{E'}_L\gamma^\mu E'_L Z_\mu - \overline{E'}^C_R\gamma^\mu E'^C_R Z_\mu)\end{aligned} \tag{4.33}$$

The above expression can be written in a neat fashion using the following relations which follow from imposing the Majorana condition of N' and general properties of the charge conjugation operator C

$$\begin{aligned}\overline{N'_L}\gamma^\mu E'^C_R &= -\overline{E'_R}\gamma^\mu N'_R, \quad \overline{E'^C_R}\gamma^\mu N'_L = -\overline{N'_L}\gamma^\mu E'_L \\ \overline{E'^C_R}\gamma^\mu E'^C_R &= -\overline{E'_R}\gamma^\mu E'_R \\ \overline{E'}\gamma^\mu E' &= \overline{E'_L}\gamma^\mu E'_L + \overline{E'_R}\gamma^\mu E'_R\end{aligned} \tag{4.34}$$

Eqn. (4.33) becomes

$$\begin{aligned}\mathcal{L}_{kinetic} &= (i\overline{N'_L}\not{\partial}N'_L + i\overline{E'^C_R}\not{\partial}E'^C_R + i\overline{E'_L}\not{\partial}E'_L) \\ &- g(\overline{E'}\gamma^\mu N' W^-_\mu + \overline{N'}\gamma^\mu E' W^+_\mu) \\ &+ e(\overline{E'}\gamma^\mu E' A_\mu) + g\cos\theta_W(\overline{E'}\gamma^\mu E' Z_\mu)\end{aligned} \tag{4.35}$$

Apart from the kinds of gauge interactions that are visible from the above equation, we need to incorporate the charged lepton gauge interactions of the Standard Model also, because of the distinctive feature of doublet and triplet charged lepton mixing in this model. The complete set of gauge interactions thus become

1. $\mathcal{L}_\gamma = e(\overline{E'}\gamma^\mu E' + \overline{\boldsymbol{\ell}'}\gamma^\mu\boldsymbol{\ell}')A_\mu$
2. $\mathcal{L}_Z = \Big[g\cos\theta_W(\overline{E'}\gamma^\mu E' + \overline{\boldsymbol{\ell}'}\gamma^\mu\boldsymbol{\ell}') - \frac{g}{\cos\theta_W}(\frac{1}{2}\overline{\boldsymbol{\nu}'_L}\gamma^\mu\boldsymbol{\nu}'_L + \frac{1}{2}\overline{\boldsymbol{\ell}'_L}\gamma^\mu\boldsymbol{\ell}'_L + \overline{\boldsymbol{\ell}'_R}\gamma^\mu\boldsymbol{\ell}'_R)\Big]Z_\mu$
3. $\mathcal{L}_W = -g(\overline{E'}\gamma^\mu N' W^-_\mu + \overline{N'}\gamma^\mu E' W^+_\mu) - \frac{g}{\sqrt{2}}(\overline{\boldsymbol{\ell}'_L}\gamma^\mu\boldsymbol{\nu}'_L W^-_\mu + \overline{\boldsymbol{\nu}'_L}\gamma^\mu\boldsymbol{\ell}'_L W^+_\mu)$

4. $\mathcal{L}_{Higgs} = \left(\overline{\boldsymbol{\nu}'_L} Y_\Sigma^\dagger N'_R + \sqrt{2}\overline{\boldsymbol{\ell}'_L} Y_\Sigma^\dagger E'_R\right)\frac{h}{\sqrt{2}} + \left(\overline{N'_R} Y_\Sigma \boldsymbol{\nu}'_L + \sqrt{2}\overline{E'_R} Y_\Sigma \boldsymbol{\ell}'_L\right)\frac{h}{\sqrt{2}}$

where h is the physical Higgs field.

In the mass eigenstate basis, the photon couplings to fermions are diagonal. This can also be directly shown using the relations that arise from the unitarity of U^L. The couplings to the Z and W bosons are more complicated and explicit expressions of the interaction terms can be found in [18].

To leading order in $Y_\Sigma v M_\Sigma^{-1}$ (using the results in eqn. (4.29)), the interaction terms involving heavy triplet leptons can be written in the mass eigen-state basis as

$$\begin{aligned}
\mathcal{L}_Z &= g\cos\theta_W(\overline{E}\gamma^\mu E)Z_\mu + \frac{g}{2\cos\theta_W}\Big[\overline{\boldsymbol{\nu}}(V_{PMNS}^\dagger V_{\ell N}\gamma^\mu P_L - V_{PMNS}^T V_{\ell N}^*\gamma^\mu P_R)N\Big]Z_\mu \\
&+ \frac{g}{\sqrt{2}\cos\theta_W}(\overline{E}V_{\ell N}^*\gamma^\mu P_L\boldsymbol{\ell} + \overline{\boldsymbol{\ell}}V_{\ell N}\gamma^\mu P_L E)Z_\mu \\
\mathcal{L}_W &= -g(\overline{E}\gamma^\mu N W_\mu^- + \overline{N}\gamma^\mu E W_\mu^+) \\
&- \frac{g}{\sqrt{2}}(V_{\ell N}\overline{\boldsymbol{\ell}}\gamma^\mu P_L N W_\mu^- + V_{\ell N}^*\overline{N}\gamma^\mu P_L\boldsymbol{\ell}W_\mu^+) \\
&- g(V_{\ell N}V_{PMNS}^\dagger\overline{E}\gamma^\mu P_R\boldsymbol{\nu}W_\mu^- + V_{\ell N}^* V_{PMNS}^T\overline{\boldsymbol{\nu}}\gamma^\mu P_R E W_\mu^+) \\
\mathcal{L}_{Higgs} &= \frac{g}{2M_W}\Big[\overline{\boldsymbol{\nu}}(V_{PMNS}^\dagger V_{\ell N}M_N^{diag}P_R + V_{PMNS}^T V_{\ell N}^* P_L M_N^{diag})N \\
&+ \sqrt{2}(V_{\ell N}\overline{\boldsymbol{\ell}}M_E^{diag}P_R E + V_{\ell N}^*\overline{E}P_L M_E^{diag}\boldsymbol{\ell})\Big]h
\end{aligned} \tag{4.36}$$

where $V_{\ell N} = -\frac{Y_\Sigma^\dagger v M_\Sigma^{-1}}{\sqrt{2}}$, $M_W = gv/2$ is the mass of the W-boson and M_N^{diag},M_E^{diag} are the eigen-mass matrices for N' and E' respectively. P_L and P_R are the chirality projection operators previously denoted by L and R respectively, written in a different convention here to avoid confusion with the Standard Model lepton doublet which is also denoted by L_L.

As already mentioned, the presence of these gauge interactions provides an advantage over the other two realizations of see-saw, because of the strength of coupling being governed by gauge couplings, contrary to Yukawa couplings in the type I and type II scenario. From the above expressions, the allowed decay modes can also be figured out. The electroweak production of the triplets at the LHC through proton-proton collisions

$$pp \to E^\pm N, \quad E^+E^-$$

will leave a bunch of multi-lepton final states followed by the triplet decays

$$E^{\pm} \rightarrow \nu_i W^{\pm} / \ell_i^{\pm} Z / \ell_i^{\pm} h$$
$$N \rightarrow \ell_j^{\pm} W^{\mp} / \nu_j Z / \nu_j h$$

Of these, decays into trilepton seem a promising channel [20] for the triplet lepton search. In trilepton final states, one of the three leptons necessarily result from the decay of a W or a Z boson, and hence its flavor is not related to the lepton triplet couplings.

In the study of these decays of E and N into Standard Model particles, the interaction matrix $V_{\ell N}$ plays an important role, as can be seen from the interaction Lagrangians obtained in eqn. (4.36). So, knowledge about the structure and constraints on $V_{\ell N}$ is very crucial. These and many more essential points regarding the phenomenology of the type III see-saw alongwith test possibilities at the LHC forms the subject matter of Chapter 5.

Chapter 5

See-saw at the LHC: discovery potential and model discrimination

In the previous chapters, I discussed in detail about the theoretical framework of the three realizations of the see-saw mechanisms. Although there are several features innate to each of these mechanisms, the main ingredients can be summarized as

<u>*Type I see-saw*</u>: Singlet right-handed (RH) neutrinos, ν_R are introduced which transform as $(1, 1, 0)$ under the SM gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. The ν_R's do not have SM gauge interactions. The light neutrino masses are given by $m_\nu \sim y_\nu^2 v^2/M_R$, where $v = 246$ GeV, y_ν is the Yukawa coupling and M_R is the RH neutrino mass which sets the scale, Λ of New-Physics (NP). For $y_\nu \sim \mathcal{O}(1)$, M_R should be of the order $10^{14} - 10^{16}$ GeV to reproduce the small neutrino masses of order eV or smaller. However, if the Yukawa couplings are of the order of electron Yukawa couplings or smaller, then M_R can be at the TeV scale.

<u>*Type II see-saw*</u>: Triplet Higgs representation Δ transforming under $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ as $(1, 3, 2)$. Neutrino masses are given by $m_\nu \approx Y_\nu v_t$, where v_t is the VEV of the neutral component of Δ. The doublet-triplet interaction term in the potential contains a parameter μ which after electro-weak symmetry breaking (EWSB) connects to v_t as $v_t \sim \mu v_d^2/M_\Delta^2$, where M_Δ is the mass of the triplet. Thus, the NP scale, Λ in this case is M_Δ^2/μ. If $Y_\nu \sim \mathcal{O}(1)$ and $\mu \sim M_\Delta$, then $\Lambda \sim 10^{14} - 10^{15}$ GeV. A TeV scale for M_Δ is however possible for small Y_ν.

Type III see-saw: Triplet lepton representations $\Sigma_{L/R}$ transforming under $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ as $(1,3,0)$ are introduced. The resulting mass matrix for neutrinos has the same form as that in Type I see-saw, $m_\nu \sim v^2 Y_\Sigma^2 / M_\Sigma$, where Y_Σ is the triplet Yukawa coupling and M_Σ is the triplet mass. The NP scale in this case is replaced by M_Σ. A TeV scale for M_Σ is once again possible.

In the absence of sufficient and credible experimental data, it is difficult to say which if these mechanisms (if any) are correct. Hence, it is extremely essential to look for positive signals in experiments that would validate these mechanisms.

5.1 Multi-lepton signals at the LHC

The Large Hadron Collider (LHC) at CERN has ushered in a new era of research in physics by bringing to us the TeV energy scale. With the 2012 discovery of a possible candidate for the Higgs boson by the ATLAS [21,22] and CMS [23,24] collaborations, the LHC has stimulated hopes for discoveries of some fundamental new physics beyond the Standard Model of elementary particle interactions.

However, the energy scale at which the LHC operates is much higher than the mass scale of the neutrinos. So, it is not possible to directly test light neutrino masses. But a feasible generic idea would be to produce the see-saw messengers (heavy degrees of freedom) as predicted by the different models. If their masses are of the Terascale, then they are accessible to the LHC and hence indirect signals of see-saw may be obtained.

In accelerator-based experiments, neutrinos in the final state are undetectable by the detectors, leading to the so-called 'missing-energy' and thus missing lepton numbers as well. It thus becomes essential to look for charged leptons in the final state. The production of see-saw messengers and possible signals associated with them is a well-researched subject. In the current discussion, I review the possibilities of discriminating see-saw models by probing the multi-lepton signals obtained through detailed studies at the level of fast simulations. This approach provides certain advantages

1. Multi-lepton signals may provide early discoveries at the LHC, which form the logical point of interest because luminosities and energies of 300 fb^{-1} and 15 TeV will not be available in recent times (expected by 2023).

2. Lepton signals are clean and most convenient for signal classification. Multi-lepton signals have an advantage over dilepton signals (as will be obvious from the subsequent discussions) in the sense that dilepton signals serve as indicators for various other processes like production of heavy singlet neutrinos N_i, heavy N_i's with new gauge interactions and so makes model-discrimination difficult.

It should be mentioned here that there are several other models like LR gauge-symmetric theories that include new gauge bosons like W_R [25-28] and Z' [29,30], SO(10) SUSY grand unification [31-33] and other SU(5) models, theories with extra dimensions etc. which explain production of neutrino masses without the constraints that we will be dealing with in the minimal see-saw models. In that way, they offer better opportunities for discovery at the LHC. However, in the current review, a conservative approach is taken in that we do not take into account such new interactions and rely on the minimal see-saw mechanisms for positive signals.

5.2 Type I see-saw

The theory of the type I see-saw was discussed in Chapter 2. In this section, I will focus on heavy neutrino interactions with the leptons of Standard Model, by taking into consideration only the lightest of the heavy neutrinos, N.

5.3 Heavy neutrino interactions

The gauge interactions involving the heavy right-handed neutrino N are given by

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}}(\overline{\boldsymbol{\ell}'}_L \gamma^\mu \boldsymbol{\nu}'_L W^-_\mu + \overline{\boldsymbol{\nu}'}_L \gamma^\mu \boldsymbol{\ell}'_L W^+_\mu) \tag{5.1}$$

$$\mathcal{L}_Z = -\frac{g}{2\cos\theta_W}\overline{\boldsymbol{\nu}'}_L \gamma^\mu \boldsymbol{\nu}'_L Z_\mu \tag{5.2}$$

$$\mathcal{L}_H = -\frac{1}{\sqrt{2}}(\overline{\boldsymbol{\nu}'}_L Y N'_R + \overline{N'}_R Y^\dagger \boldsymbol{\nu}'_L)h \tag{5.3}$$

In terms of mass-eigenstates, these can be written (to leading order in Yv/m_N) as

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}}(V_{\ell N}\overline{\boldsymbol{\ell}}\gamma^\mu P_L N W^-_\mu + V^*_{\ell N}\overline{N}\gamma^\mu P_L \boldsymbol{\ell} W^+_\mu) \tag{5.4}$$

$$\mathcal{L}_Z = -\frac{g}{2\cos\theta_W}(V_{\ell N}\overline{\boldsymbol{\nu}}\gamma^\mu P_L N + V^*_{\ell N}\overline{N}\gamma^\mu P_L \boldsymbol{\nu})Z_\mu \tag{5.5}$$

$$\mathcal{L}_H = -\frac{g m_N}{2M_W}(V_{\ell N}\overline{\boldsymbol{\nu}} P_R N + V^*_{\ell N}\overline{N} P_L \boldsymbol{\nu})h \tag{5.6}$$

$V_{\ell N}$ represents the mixing between charged leptons ℓ and the heavy neutrino N.

$$V_{\ell N} \simeq \frac{Y_{\ell N} v}{\sqrt{2} m_N} \tag{5.7}$$

Using the Majorana nature of N and $\boldsymbol{\nu}$, we can write $\overline{N}\gamma^\mu P_L \boldsymbol{\nu} = -\overline{\boldsymbol{\nu}}\gamma^\mu P_R N$ so that the neutral and scalar interactions can be rewritten as

$$\mathcal{L}_Z = -\frac{g}{2\cos\theta_W}\overline{\boldsymbol{\nu}}\gamma^\mu (V_{\ell N} P_L - V_{\ell N}^* P_R) N Z_\mu \tag{5.8}$$

$$\mathcal{L}_H = -\frac{g m_N}{\sqrt{2} M_W}\overline{\boldsymbol{\nu}}(V_{\ell N} P_R + V_{\ell N}^* P_L) N h \tag{5.9}$$

The light neutrino masses m_ν are of the order $Y^2 v^2/2m_N$ and hence $V_{\ell N} \sim \sqrt{\frac{m_\nu}{m_N}}$. Even for $m_N \sim 100$ GeV, which is well within the reach of the LHC, $V_{\ell N} \sim 10^{-6}$ to reproduce $m_\nu \sim 0.1$ eV. It is quite evident from eqns. (5.4), (5.5) and (5.6) that the gauge interactions are dependent on $V_{\ell N}$. In the cross-sections, the dependence appears as $\mid V_{\ell N} \mid^2 \sim \mathcal{O}(10^{-12})$ which makes observation of heavy neutrino signals very difficult. Apart from this model-dependent constraint, there are several other constraints imposed on $V_{\ell N}$ by experiments. These can be categorized under

Electroweak precision measurements: These constraints are obtained from universality tests, unitarity measurements of the CKM matrix and the invisible decay width of Z-boson. The current experimental data [34] constrains the $V_{\ell N}$ $(\ell = e, \mu, \tau)$ values as

$$\mid V_{eN} \mid^2 \leq 0.0030, \quad \mid V_{\mu N} \mid^2 \leq 0.0032, \quad \mid V_{\tau N} \mid^2 \leq 0.0062 \tag{5.10}$$

Non-observation of lepton-flavor violating (LFV) processes like $\mu \to e\gamma$, $\mu \to ee^+e^-$ and $\mu - e$ conversion in nuclei etc. also lead to certain constraints involving two charged leptons.[35-37]

$$\mid V_{eN} V_{\mu N}^* \mid \leq 0.0001, \quad \mid V_{eN} V_{\tau N}^* \mid \leq 0.01, \quad \mid V_{\mu N} V_{\tau N}^* \mid \leq 0.01 \tag{5.11}$$

Although these bounds are not as stringent as the earlier ones, the first one in the above equation is important as it suggests that a heavy neutrino cannot have substantial mixings with the electron and muon simultaneously.

Neutrinoless double-beta decay $0\nu\beta\beta$: In the $0\nu\beta\beta$ process, the contribution of the heavy neutrino N to the decay amplitude is described by the standard Majorana exchange between two β-decaying neutrons.

The best experimental lower bound on $0\nu\beta\beta$-decay half-life was obtained [38] for ^{76}Ge.

$$T^{0\nu}_{1/2}(^{76}\mathrm{Ge}) \geq 1.9 \times 10^{25} yrs \tag{5.12}$$

For the light-heavy neutrino case that we are dealing with, this translates into

$$\mid V_{eN} \mid^2 \frac{1}{m_N} \leq 5 \times 10^{-8}\mathrm{GeV}^{-1} \tag{5.13}$$

This is a highly stringent constraint and makes the observation of like-sign dielectrons unlikely.

5.4 See-saw I signals

The following partonic process is considered

$$q\bar{q}' \rightarrow W^* \rightarrow \ell^{\pm} N \tag{5.14}$$

for single heavy neutrino production. The cross-section depends both on m_N and the small mixing $V_{\ell N}$. Partial widths for the decays of heavy Majorana singlets are

$$\begin{aligned}
\Gamma(N \rightarrow \ell^- W^+) = \Gamma(N \rightarrow \ell^+ W^-) &= \frac{g^2}{64\pi} \mid V_{\ell N} \mid^2 \frac{m_N^3}{M_W^2}\Big(1 - \frac{M_W^2}{m_N^2}\Big)\Big(1 + \frac{M_W^2}{m_N^2} - 2\frac{M_W^4}{m_N^4}\Big) \\
\Gamma(N \rightarrow \nu Z) &= \frac{g^2}{64\pi \cos^2\theta_W} \mid V_{\ell N} \mid^2 \frac{m_N^3}{M_Z^2}\Big(1 - \frac{M_Z^2}{m_N^2}\Big)\Big(1 + \frac{M_Z^2}{m_N^2} - 2\frac{M_Z^4}{m_N^4}\Big) \\
\Gamma(N \rightarrow \nu h) &= \frac{g^2}{64\pi} \mid V_{\ell N} \mid^2 \frac{m_N^3}{M_W^2}\Big(1 - \frac{M_h^2}{m_N^2}\Big)^2
\end{aligned} \tag{5.15}$$

For $m_N < M_W$, these two-body decays are not possible and N decays into three fermions mediated by off-shell bosons (W^-, W^+, h). As can be seen from the above equations, the branching fractions for individual final states are in the ratios $\mid V_{eN} \mid^2 : \mid V_{\mu N} \mid^2 : \mid V_{\tau N} \mid^2$. On the other hand, the total branching ratios for each of the four channels is independent of $V_{\ell N}$ and is determined only by m_N, M_W, M_Z and M_h. As already stated, production cross-sections are highly suppressed by the small mixings $V_{\ell N}$ even for TeV scale m_N. The decay products of the heavy neutrino are thus not very energetic and the SM backgrounds must be taken into consideration in any analysis regarding detection of heavy neutrino signals.

Among the possible final states suggested by the decay eqns. (5.15), only charged current decays give final states that may be observed in principle. Single heavy neutrino production processes like

$$q\bar{q} \to Z^* \to \nu N, \quad gg \to h^* \to \nu N \tag{5.16}$$

give $\ell^\pm$ and $\ell^+\ell^-$ final states which are unobservable due to the huge backgrounds. Pair production of heavy neutrinos in

$$q\bar{q} \to Z^* \to NN \tag{5.17}$$

are suppressed by $\mid V_{\ell N} \mid^4$ and hence negligible. In the following, final states of different lepton multiplicity are examined for the observation of a positive signal with possible means for distinguishing from the other types of see-saw. These are the results of simulation studies [39] with the heavy neutrino mass, m_N assumed to be ~ 100 GeV for which cross-section are relatively large.

1. **$\ell^\pm\ell^\pm$ final state**: They are produced from the Lepton-Number-Violating (LNV) decay of the heavy neutrino and subsequent hadronic W decay

$$q\bar{q}' \to \ell^\pm N \to \ell^\pm\ell^+W^-, \quad W \to q\bar{q}' \tag{5.18}$$

or leptonic W decay when the lepton is missed. The observation of these signals are severely limited by SM background processes. For eg. processes having b and/or $\bar{b}$ quarks in the final state like $t\bar{t}nj$ (n is the number of jets) with semi-leptonic decay of the $t\bar{t}$ pair and $Wb\bar{b}nj$ with leptonic decay of W. Additional like-sign μ-leptons result from the decay of a b or $\bar{b}$ quark. Only a tiny fraction of such decays produce isolated muons with sufficiently high transverse momenta, p_T. Both $t\bar{t}nj$ and $Wb\bar{b}nj$ cross-sections are large and hence these backgrounds are larger than those with two weak gauge bosons. Even with imposition of large cuts in transverse momentum, these SM backgrounds are non-negligible. Thus, heavy neutrino signals are very difficult to observe in this channel.

2. **$\ell^\pm$ $\ell^\pm$ $\ell^\mp$ final state**: These signals are produced in charged current decay channels of the heavy neutrino, with subsequent leptonic decay of the W boson

$$\begin{aligned} \ell^+N &\to \ell^+\ell^-W^+ \to \ell^+\ell^-\ell^+\bar{\nu} \\ \ell^+N &\to \ell^+\ell^+W^- \to \ell^+\ell^+\ell^-\nu \end{aligned} \tag{5.19}$$

τ leptonic decays also have small additional contributions. This final state is much cleaner than the like-sign dilepton one. However, getting a signature of new physics is very difficult even with modifications which indicate that heavy neutrino singlets require dedicated searches, optimized for their detection.

5.5 Viewpoint

- Heavy neutrino signals are limited by the stringent constraints imposed on the mixing parameters $V_{\ell N}$ by precision measurements.
- For the heavy neutrino mass range which is accessible at the LHC, the SM backgrounds are large and heavy neutrino production cross-sections are very small thereby making these singlets really difficult to observe.
- The trilepton channel is somewhat better than the like-sign dilepton one.
- Heavy neutrino signals are characterized by low transverse momenta and by a broad like-sign invariant mass distribution which does not have any peaks or long tails. This makes the distinction from type II and type III see-saw signals possible.
- Scalar triplets (type II) and fermion triplets (type III) have a lot of other final states that are not endemic to heavy neutrino singlets and hence the discrimination is easier is the event of a positive signal.

5.6 Type II see-saw

In chaper 3, the theory of the type II see-saw mechanism was explained thoroughly. The charged lepton couplings with the scalar triplet with possible phenomenological consequences were also discussed. In this discussion, I follow the same approach as earlier "*finding possibility of detection of heavy Higgs triplets through multi-lepton signals*". Before proceeding towards that attempt, I elaborate the physical scalar spectrum of the type II see-saw which in turn will validate the general assumptions that are made throughout this review.

5.7 The physical Higgs spectrum

The physical scalar spectrum consists of 7 Higgs bosons designated by $H^{\pm\pm}$,$H^{\pm}$, H^0,A^0 and h^0. The physical states and their masses are given by

1. *Doubly charged Higgs*: These are purely triplet states $H^{\pm\pm} = \delta^{\pm\pm}$. The mass is given by

$$m^2_{H^{\pm\pm}} = \frac{\mu v_d^2}{\sqrt{2}v_t} - \frac{\lambda_4 v_d^2}{2} - \lambda_3 v_t^2 \tag{5.20}$$

It is evident from Table 2 and section 3.4 of Chapter 3 that v_t is much smaller than v_d to account for the small neutrino masses and also $\delta\rho < 0$. So, neglecting the terms proportional to v_t^2, we get

$$m^2_{H^{\pm\pm}} \simeq \frac{\mu v_d^2}{\sqrt{2}v_t} - \frac{\lambda_4 v_d^2}{2} \tag{5.21}$$

Moreover, in our discussion, we are looking for heavy Higgs triplets so that terms proportional to λ_i's (i=1,2,3,4) are neglected. These are the two assumptions made in the current analysis. Under these assumptions

$$m^2_{H^{\pm\pm}} \simeq \frac{\mu v_d^2}{\sqrt{2}v_t} = M_\Delta^2 \qquad \text{(eqn. (3.17))} \tag{5.22}$$

2. *Singly charged Higgs*: These are a mixture of doublet and triplet states. The physical states are given by

$$\begin{aligned} H^{\pm} &= -\epsilon_\beta \frac{\sqrt{2}v_t}{\sqrt{v_d^2 + 2v_t^2}}\phi^{\pm} + \epsilon_\beta \frac{v_d}{\sqrt{v_d^2 + 2v_t^2}}\delta^{\pm} \\ &\simeq \epsilon_\beta \delta^{\pm}, \qquad \text{for} \quad v_t << v_d \end{aligned} \tag{5.23}$$

where $\epsilon_\beta = \pm 1$ is an overall sign-factor. Thus, $H^{\pm}$ is mainly constituted by the triplet Higgs component. The mass is given by

$$\begin{aligned} m^2_{H^{\pm}} &= \frac{\mu v_d^2}{\sqrt{2}v_t}(1 + 2\frac{v_t^2}{v_d^2}) - \frac{\lambda_4 v_d^2}{4}(1 + 2\frac{v_t^2}{v_d^2}) \\ &\simeq \frac{\mu v_d^2}{\sqrt{2}v_t}, \qquad \text{for} \quad v_t << v_d, \lambda_4 = 0 \end{aligned} \tag{5.24}$$

From eqns. (5.22) and (5.24), it can be seen that the doubly and singly charged scalars are nearly degenerate in mass. However, if λ_i's are not zero, then there is a mass splitting between $H^{\pm\pm}$ and $H^{\pm}$.

$$m^2_{H^{\pm\pm}} - m^2_{H^{\pm}} = -\frac{\lambda_4 v_d^2}{4} \tag{5.25}$$

The sign of λ_4 determines the relative magnitudes of $H^{\pm\pm}$ and $H^{\pm}$. From [40], $|\Delta M| \approx 540$ MeV, which is much smaller and generally do not affect the production processes of our concern. In case of transitions between two heavy triplet Higgs bosons via SM gauge interactions like $H^{\pm\pm} \to H^{\pm}W^{\pm *}$, $H^{\pm} \to H^0 W^{\pm *}$, this mass-splitting may have sizable contributions. In the current analysis however, this effect is neglected.

3. <u>*Neutral fields*</u>: These are classified under CP-even and CP-odd fields depending on their transformation properties under CP. From

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{v_d+h+iZ_1}{\sqrt{2}} \end{pmatrix}, \qquad \Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \frac{v_t+\xi^0+iZ_2}{\sqrt{2}} & -\delta^+/\sqrt{2} \end{pmatrix} \tag{5.26}$$

it is clear that (h, ξ^0) are CP-even and (Z_1, Z_2) are CP-odd fields. The physical CP-even fields denoted by h^0 and H^0 are

$$h^0 = \cos\alpha h + \sin\alpha\xi^0, \qquad \tan 2\alpha \simeq \frac{4v_t}{v_d} \tag{5.27}$$

Thus, α is very small indicating that $h^0 \simeq h$ is mostly SM doublet-like Higgs and $H^0 \simeq \xi^0$ is mainly triplet-like. The masses are related as

$$\begin{aligned} m_{h^0}^2 + m_{H^0}^2 &= \frac{\lambda v_d^2}{2} + \frac{\mu v_d^2}{\sqrt{2}v_t} + 2(\lambda_2+\lambda_3)v_t^2 \\ &\simeq \frac{\lambda v_d^2}{2} + \frac{\mu v_d^2}{\sqrt{2}v_t} \end{aligned} \tag{5.28}$$

Since h^0 is doublet-like, so assuming $m_{h^0}^2 = \frac{\lambda v_d^2}{2}$ we get $m_{H^0}^2 \simeq \mu v_d^2/\sqrt{2}v_t$.

The physical CP-odd field is A^0. It is given by

$$\begin{aligned} A^0 &= -\epsilon_{\beta'}\frac{2v_t}{\sqrt{v_d^2+4v_t^2}}Z_1 + \epsilon_{\beta'}\frac{v_d}{\sqrt{v_d^2+4v_t^2}}Z_2 \\ &\simeq \epsilon_{\beta'}Z_2 \end{aligned} \tag{5.29}$$

where $\epsilon_{\beta'} = \pm 1$ is once again an overall sign factor. A^0 is thus mainly of triplet nature. Its mass is given by

$$m_{A^0}^2 = \frac{\mu(v_d^2+4v_t^2)}{\sqrt{2}v_t} \simeq \frac{\mu v_d^2}{\sqrt{2}v_t} \tag{5.30}$$

To summarize, the six mass eigenstates $H^{\pm\pm}$, $H^{\pm}$, H^0 and A^0 are essentially composed of triplet fields whereas h^0 is mainly of doublet Higgs. In our approximation

$$m_{H^{\pm\pm}}^2 \simeq m_{H^{\pm}}^2 \simeq m_{A^0}^2 \simeq m_{H^0}^2 \simeq \frac{\mu v_d^2}{\sqrt{2}v_t} = M_\Delta^2 \tag{5.31}$$

5.8 Heavy scalar triplet interactions

As mentioned in section 3.3, the charged lepton couplings with the triplet are given by

1. Singly charged scalar:

$$\mathcal{L}_{+\ell} = \sum_{\alpha,\beta} -\frac{Y_{\alpha\beta}}{\sqrt{2}}(\boldsymbol{\nu}^T_{\alpha L}C^\dagger\delta^+\boldsymbol{\ell}_{\beta L} + \boldsymbol{\ell}^T_{\alpha L}C^\dagger\delta^+\boldsymbol{\nu}_{\beta L}) + h.c \tag{5.32}$$

2. Doubly charged scalar:

$$\mathcal{L}_{++\ell} = \sum_{\alpha,\beta} Y_{\alpha\beta}\boldsymbol{\ell}^T_{\alpha L}C^\dagger\delta^{++}\boldsymbol{\ell}_{\beta L} + h.c \tag{5.33}$$

where $Y_{\alpha\beta} = \Big[V^*_{PMNS}diag(m_1, m_2e^{i\varphi_1}, m_3e^{i\varphi_2})V^\dagger_{PMNS}\Big]_{\alpha\beta}/\sqrt{2}v_t$. In our approximation, δ^+ and δ^{++} in eqns. (5.32) and (5.33) can be replaced by H^+ and H^{++} respectively.

The gauge interactions that mediate scalar triplet pair production processes are

$$\begin{aligned}\mathcal{L}_W =& - ig\Big[(\partial^\mu H^{--})H^+ - H^{--}(\partial^\mu H^+)\Big]W^+_\mu \\ &- ig\Big[(\partial^\mu H^-)H^{++} - H^-(\partial^\mu H^{++})\Big]W^-_\mu\end{aligned} \tag{5.34}$$

$$\begin{aligned}\mathcal{L}_Z =& \frac{ig}{\cos\theta_W}(1 - 2\sin^2\theta_W)\Big[(\partial^\mu H^{--})H^{++} - H^{--}(\partial^\mu H^{++})\Big]Z_\mu \\ &- \frac{ig}{\cos\theta_W}\sin^2\theta_W\Big[(\partial^\mu H^-)H^+ - H^-(\partial^\mu H^+)\Big]Z_\mu\end{aligned} \tag{5.35}$$

$$\begin{aligned}\mathcal{L}_\gamma =& i2e\Big[(\partial^\mu H^{--})H^{++} - H^{--}(\partial^\mu H^{++})\Big]A_\mu \\ &+ ie\Big[(\partial^\mu H^-)H^+ - H^-(\partial^\mu H^+)\Big]A_\mu\end{aligned} \tag{5.36}$$

Constraints on the triplet parameters are much less important because these new scalars can be produced at the LHC by unsuppressed gauge couplings, unlike heavy neutrino singlets. As discussed in section 3.5, electroweak precision measurement data on the ρ-parameter restricts $v_t < 2.5$ GeV which is much less stringent than the ones obtained from the smallness of neutrino masses (Table 2). As we will see in section 5.9, the relative values of $Y_{\alpha\beta}$ and v_t govern the decays of the scalars.

Some constraints on $Y_{\alpha\beta}$ arise from four-fermion processes like $\mu^- \to e^+e^-e^-$, $\tau \to 3\ell$ and LFV processes. These constraints are of the order 10^{-2} or larger except [19]

$$\mid Y_{ee}Y^*_{e\mu} \mid < 2.4 \times 10^{-5} \tag{5.37}$$

which is obtained from $\mu^- \to e^+e^-e^-$. These are automatically satisfies for $Y_{\alpha\beta} \sim \mathcal{O}(10^{-3})$.

5.9 Heavy Higgs decays

1. Doubly charged Higgs decays: The possible decays of $H^{\pm\pm}$ are the ones into like-sign dileptons or dibosons.

$$H^{\pm\pm} \to \ell^{\pm}_{\alpha}\ell^{\pm}_{\beta}, \qquad H^{\pm\pm} \to W^{\pm}W^{\pm}, \quad \ell_{\alpha,\beta} = e, \mu, \tau \tag{5.38}$$

The partial decay width for the like-sign dilepton decay is given by

$$\begin{aligned}\Gamma(H^{\pm\pm} \to \ell^{\pm}_{\alpha}\ell^{\pm}_{\beta}) &= \frac{S m_{H^{\pm\pm}} \mid Y_{\alpha\beta} \mid^2}{8\pi} \\ &= \frac{S m_{H^{\pm\pm}} \mid M^{\nu}_{\alpha\beta} \mid^2}{16\pi v_t^2}\end{aligned} \tag{5.39}$$

where $S = 1(2)$ for $\alpha = \beta$ $(\alpha \neq \beta)$.

The decay widths for $H^{\pm\pm} \to W^{\pm}_L W^{\pm}_L$ and $H^{\pm\pm} \to W^{\pm}_T W^{\pm}_T$, where L and T denote longitudinal and transverse polarizations of the W gauge boson, are given by

$$\Gamma(H^{\pm\pm} \to W^{\pm}_T W^{\pm}_T) = \frac{2M_W^4 v_t^2}{\pi v_d^4 m_{H^{\pm\pm}}}\Big(1 - \frac{4M_W^2}{m^2_{H^{\pm\pm}}}\Big)^{1/2} \tag{5.40}$$

$$\Gamma(H^{\pm\pm} \to W^{\pm}_L W^{\pm}_L) = \frac{v_t^2 m^3_{H^{\pm\pm}}}{4\pi v_d^4}\Big(1 - \frac{4M_W^2}{m^2_{H^{\pm\pm}}}\Big)^{1/2}\Big(1 - \frac{2M_W^2}{m^2_{H^{\pm\pm}}}\Big)^2 \tag{5.41}$$

so that

$$\Gamma(H^{\pm\pm} \to W^{\pm}W^{\pm}) = \frac{v_t^3 m^3_{H^{\pm\pm}}}{4\pi v_d^4}\Big(1 - \frac{4M_W^2}{m^2_{H^{\pm\pm}}}\Big)^{1/2}\Big(1 - \frac{4M_W^2}{m^2_{H^{\pm\pm}}} + \frac{12M_W^4}{m^4_{H^{\pm\pm}}}\Big) \tag{5.42}$$

The relative decay rate is

$$\frac{\Gamma(H^{\pm\pm} \to \ell^{\pm}_{\alpha}\ell^{\pm}_{\beta})}{\Gamma(H^{\pm\pm} \to W^{\pm}W^{\pm})} \approx \Big(\frac{m_\nu}{m_{H^{\pm\pm}}}\Big)^2\Big(\frac{v_d}{v_t}\Big)^4 \tag{5.43}$$

where I have assumed that the basis we are working is the one in which neutrino mass matrix is diagonal.

For $\left(\frac{m_\nu}{m_{H^{\pm\pm}}}\right) \sim \frac{1eV}{1TeV} = (10^{-12})$, we have the decay rates comparable at $v_t \approx 10^{-4}$ GeV. Thus, for $v_t < 10^{-4}$ GeV, the dominant decay mode is the one into like-sign dileptons.

As our point of interest is in multi-lepton final states, so v_t lies in this range in our analysis. Also, the assumptions initially made in this chapter work well with this range of v_t.

2. Singly charged Higgs decays: In case of singly charged Higgs boson, the possible decays are

$$H^\pm \to \ell^+\overline{\nu}/\ell^-\nu, \quad H^\pm \to W^\pm h^0, \quad H^\pm \to W^\pm Z, \quad H^\pm \to t\bar{b}/\bar{t}b \tag{5.44}$$

The partial decay width for the first channel is

$$\Gamma(H^\pm \to \ell^\pm\overline{\nu}/\nu) = \frac{m_{H^\pm} \mid M^\nu \mid^2}{16\pi v_t^2} \tag{5.45}$$

For the other three channels, the approximate decay widths are

$$\begin{aligned}
\Gamma(H^\pm \to W^\pm Z) &\simeq \frac{v_t^2 m_{H^\pm}^3}{8\pi v_d^4} \\
\Gamma(H^\pm \to W^\pm h^0) &\simeq \frac{v_t^2 m_{H^\pm}^3}{8\pi v_d^4} \\
\Gamma(H^\pm \to t\bar{b}/\bar{t}b) &\simeq \frac{3v_t^2 m_t^2 m_{H^\pm}}{4\pi v_d^4}\left(1 - \frac{m_t^2}{m_{H^\pm}^2}\right)^2
\end{aligned} \tag{5.46}$$

For small values of v_t, the first decay mode is the dominant one while the latter three are suppressed.

We are not considering the decays of neutral scalar fields since they do not give rise to the final states that we are interested in.

5.10 See-saw II signals

Three channels in which the members of the scalar triplet can be produced in hadron colliders are

$$\begin{aligned}
q\bar{q} &\to Z^*/\gamma \to H^{++}H^{--} \\
q\bar{q}' &\to W^* \to H^{\pm\pm}H^\mp \\
q\bar{q} &\to Z^*/\gamma^* \to H^+H^-
\end{aligned} \tag{5.47}$$

The cross-sections for these processes depend only on the masses of these scalars. The second production channel is the largest source of doubly charged scalars [42] with a cross-section almost twice that for $H^{++}H^{--}$.

I now classify the signals based on lepton multiplicity of the final state and review the potential of each such state in the detection of heavy Higgs triplets based on simulation studies [39]. The common mass of the triplet is $M_\Delta \sim 300$ GeV.

1. **$\ell^+\ell^+\ell^-\ell^-$ final state**: This is a good channel for the observation of $H^{++}H^{--}$ production due to practical absence of the SM background. However, the triplet signals are smaller than in other final states due to several reasons. Firstly, the production channel $H^{\pm\pm}H^{\mp}$, which has a cross-section almost twice that of $H^{++}H^{--}$, does not contribute to this final state. Secondly, the presence of more number of charged leptons in the final state significantly reduces the branching ratio compared to those having smaller number of leptons. Finally, since all four leptons have to be isolated (with p_T above a certain threshold and within the detector acceptance), the detection efficiency is lower than in channels having lower lepton multiplicity.

2. **$\ell^\pm\ell^\pm\ell^\mp$ final state**: This is a highly promising channel for scalar triplet discovery. It has small SM backgrounds and receives contribution from $H^{\pm\pm}H^{\mp}$ production and hence much larger signals are obtained. This in turn allows for an early discovery of $H^{\pm\pm}$ at the LHC which forms our principal target. The contribution to this state from the $H^{++}H^{--}$ production channel is also larger because of the smaller number of charged leptons. Once again, the doubly charged scalars can be identified as a peak in the invariant dilepton mass distribution. Studies show that the peak is more pronounced than in the four lepton final state thereby making the $H^{\pm\pm}$ signal much easier.

3. **$\ell^\pm\ell^\pm$ final state**: These states are produced when one doubly charged scalar decays into two charged leptons while the accompanying scalar decays into neutrinos, τ jets or charged leptons that are missed by the detector. Although like-sign dileptons are common to all the three types of see-saw, a distinctive feature of the type II see-saw is that the like-sign invariant mass spectrum exhibits a peak at $M_{H^{\pm\pm}}$ which is produced when the doubly charged scalars decay directly to light charged leptons e,μ. SM backgrounds are larger compared to the previous modes but signal significance is comparable to the one achieved in four-lepton final state. A disadvantage of this final state, however, is that a full event reconstruction with two competing signal processes and several missing particles is much more involved.

4. **$\boldsymbol{\ell^+\ell^-} j_\tau$ final state**: Opposite sign dilepton backgrounds are huge at the LHC mainly coming from the $t\bar{t}nj$ and Z^*/γ^*nj decays. Hence, observation of scalar triplets in the $\ell^+\ell^-$ channel is virtually impossible. However, using the transverse momentum of the leading charged lepton as a discriminant instead of the dilepton invariant mass, the contribution of the scalar triplets to the $\ell^+\ell^- j_\tau$ final state can be detected in the form of a long tail in the transverse momentum distribution.

5. **$\boldsymbol{\ell^\pm} j_\tau j_\tau j_\tau$ final state** : The huge cross-section for Wnj production makes the signals in eqn. (5.47) unobservable in final states with only one charged lepton.

5.11 Viewpoint

- $H^{\pm\pm}H^\mp$ production channel is the dominant one for production of doubly charged scalars.
- The trilepton channel is the best for finding evidence of doubly-charged scalars at a lower luminosity this promising early discoveries.
- The like-sign dilepton channel has comparable sensitivity but requires a much more involved event reconstruction.
- Channels with oppositely charged dileptons or single leptons are very difficult to observe because of the large SM backgrounds.

5.12 Type III see-saw

Chapter 4 dealt with the type III see-saw mechanism in detail. In this section, I will rewrite the gauge interactions by taking into consideration the lightest heavy neutrino, N and then move on to analyze the plenitude of signals possible in this realization of neutrino mass generation.

5.13 Gauge interactions of heavy fermion triplets

The gauge interactions in the mass-eigen basis are given by

$$\mathcal{L}_\gamma = e\overline{E}\gamma^\mu E A_\mu$$

$$\begin{aligned}\mathcal{L}_W = &-g(\overline{E}\gamma^\mu N W^-_\mu + \overline{N}\gamma^\mu E W^+_\mu) - g(V_{\ell N}\overline{E}\gamma^\mu P_R \boldsymbol{\nu} W^-_\mu + V^*_{\ell N}\overline{\boldsymbol{\nu}}\gamma^\mu P_R E W^+_\mu) \\ &- \frac{g}{\sqrt{2}}(V_{\ell N}\overline{\boldsymbol{\ell}}\gamma^\mu P_L N W^-_\mu + V^*_{\ell N}\overline{N}\gamma^\mu P_L \boldsymbol{\ell} W^+_\mu)\end{aligned} \tag{5.48}$$

$$\begin{aligned}\mathcal{L}_Z = &\, g\cos\theta_W \overline{E}\gamma^\mu E Z_\mu + \frac{g}{2\cos\theta_W}\Big[(V_{\ell N}\overline{\boldsymbol{\nu}}\gamma^\mu P_L N + V^*_{\ell N}\overline{N}\gamma^\mu P_L \boldsymbol{\nu}) Z_\mu\Big] \\ &+ \frac{g}{\sqrt{2}\cos\theta_W}(V_{\ell N}\overline{\boldsymbol{\ell}}\gamma^\mu P_L E + V^*_{\ell N}\overline{E}\gamma^\mu P_L \boldsymbol{\ell}) Z_\mu\end{aligned} \tag{5.49}$$

$$\mathcal{L}_H = \frac{g m_\Sigma}{2 M_W}(V_{\ell N}\overline{\boldsymbol{\nu}} P_R N + V^*_{\ell N}\overline{N} P_L \boldsymbol{\nu}) h + \frac{g m_\Sigma}{\sqrt{2} M_W}(V_{\ell N}\overline{\boldsymbol{\ell}} P_R E + V^*_{\ell N}\overline{E} P_L \boldsymbol{\ell}) h \tag{5.50}$$

where

$$V_{\ell N} \simeq -\frac{Y_{\ell N} v}{\sqrt{2} m_N} \tag{5.51}$$

and $m_\Sigma = m_E = m_N$ is the common triplet mass.

The Majorana nature of ν and N can be exploited to rewrite the neutral and scalar interactions as

$$\mathcal{L}_{Z\nu N} = \frac{g}{2\cos\theta_W}\overline{\boldsymbol{\nu}}\gamma^\mu (V_{\ell N} P_L - V^*_{\ell N} P_R) N Z_\mu \tag{5.52}$$

$$\mathcal{L}_{H\nu N} = \frac{g m_\Sigma}{2 M_W}\overline{\boldsymbol{\nu}}(V_{\ell N} P_R + V^*_{\ell N} P_L) N h \tag{5.53}$$

From electroweak precision measurement data [34], the constraints on $V_{\ell N}$'s are

$$\mid V_{eN} \mid^2 \leq 0.00036 \quad \mid V_{\mu N} \mid^2 \leq 0.00029 \quad \mid V_{\tau N} \mid^2 \leq 0.00073 \tag{5.54}$$

Additional constraints appear from the non-observation of LFV processes like $\mu \to e\gamma$, $\tau \to e\gamma$, $\tau \to \mu\gamma$. The bounds obtained [19,41] at 90 % CL are

$$\mid V_{eN} V^*_{\mu N} \mid \leq 1.1 \times 10^{-6}, \quad \mid V_{eN} V^*_{\tau N} \mid \leq 0.0012, \quad \mid V_{\mu N} V^*_{\tau N} \mid \leq 0.0012 \tag{5.55}$$

As can be seen from the first terms of eqns. (5.48) and (5.49), the production of triplets at LHC can take place through gauge interactions of order unity and hence the above constraints are not very relevant.

5.14 See-saw III signals

The main production channels of heavy triplet leptons at the LHC are the partonic processes

$$\begin{aligned} q\bar{q} &\to Z^*/\gamma^* \to E^+E^- \\ q\bar{q}' &\to W^* \to E^\pm N \end{aligned} \tag{5.56}$$

These production processes are dominated by gauge couplings. Since N has $I_3 = 0$ and $Y = 0$, so it does not couple to Z. Thus, neutral lepton pairs are bot produced. In the coming discussions, the small triplet mass splittings are neglected and a common mass $m_\Sigma = m_E = m_N \sim 300$ GeV is assumed.

The partial decay widths for N are already mentioned in eqns. (5.15). The decay widths for E decays are

$$\begin{aligned} \Gamma(E^+ \to \bar{\nu}W^+) &= \frac{g^2}{32\pi} \mid V_{\ell N} \mid^2 \frac{m_E^3}{M_W^2}\Big(1 - \frac{M_W^2}{m_E^2}\Big)\Big(1 + \frac{M_W^2}{m_E^2} - \frac{2M_W^4}{m_E^4}\Big) \\ \Gamma(E^+ \to \ell^+ Z) &= \frac{g^2}{64\pi\cos^2\theta_W} \mid V_{\ell N} \mid^2 \frac{m_E^3}{M_Z^2}\Big(1 - \frac{M_Z^2}{m_E^2}\Big)\Big(1 + \frac{M_Z^2}{m_E^2} - \frac{2M_Z^4}{m_E^4}\Big) \\ \Gamma(E^+ \to \ell^+ h) &= \frac{g^2}{64\pi} \mid V_{\ell N} \mid^2 \frac{m_E^3}{M_W^2}\Big(1 - \frac{M_h^2}{m_E^2}\Big) \end{aligned} \tag{5.57}$$

and similarly for the charge conjugate E^-. The total branching ratios for decays into W, Z and h do not depend on the values of $V_{\ell N}$ but only on m_E/m_N. The partial widths to different flavors are in the ratio

$$\frac{\Gamma(E^+ \to \nu_{\ell_1} W^+)}{\Gamma(E^+ \to \nu_{\ell_2} W^+)} = \frac{\Gamma(E^+ \to \ell_1 Z)}{\Gamma(E^+ \to \ell_2 Z)} = \frac{\Gamma(E^+ \to \ell_1 h)}{\Gamma(E^+ \to \ell_2 h)} = \frac{\mid V_{\ell_1 N} \mid^2}{\mid V_{\ell_2 N} \mid^2} \tag{5.58}$$

Several final states are possible even for the triplet coupling to a particular lepton flavor.

Table 3: Final states for E^+E^- production in type III see-saw picture.

	$E^+ \to \bar{\nu}W^+$	$E^+ \to \ell^+ Z$	$E^+ \to \ell^+ h$
$E^- \to \nu W^-$	$\nu\bar{\nu}W^+W^-$	$\ell^+\nu Z W^-$	$\ell^+\nu h W^-$
$E^- \to \ell^- Z$	$\ell^-\bar{\nu}W^+Z$	$\ell^+\ell^- ZZ$	$\ell^+\ell^- Zh$
$E^- \to \ell^- h$	$\ell^-\bar{\nu}hW^+$	$\ell^+\ell^- Zh$	$\ell^+\ell^- hh$

Table 4: Final states for E^+N production in type III see-saw picture.

	$E^+ \to \overline{\nu} W^+$	$E^+ \to \ell^+ Z$	$E^+ \to \ell^+ h$
$N \to \ell^- W^+$	$\ell^- \overline{\nu} W^+ W^+$	$\ell^+ \ell^- Z W^+$	$\ell^+ \ell^- h W^+$
$N \to \ell^+ W^-$	$\ell^+ \overline{\nu} W^+ W^-$	$\ell^+ \ell^+ Z W^-$	$\ell^+ \ell^+ h W^-$
$N \to \nu Z$	$\nu \overline{\nu} W^+ Z$	$\ell^+ \nu Z Z$	$\ell^+ \nu h Z$
$N \to \nu h$	$\nu \overline{\nu} h W^+$	$\ell^+ \nu h Z$	$\ell^+ \nu h h$

and similarly for $E^- N$ production.

Thus, it is fairly clear that a large number of final state signatures are possible from the decays channels. The final states considered in this review are classified in terms of lepton multiplicity.

1. **Six lepton final states**: These final states are only produced in the channel

$$E^+ E^- \to \ell^+ Z \ell^- Z, \qquad Z \to \ell^+ \ell^- \tag{5.59}$$

Although these states have practically no SM background, the signal cross-sections are very small. For $m_E \sim 300$ GeV, the overall branching ratio is calculated to be very small (=3.5×10^{-4}). This state is relevant only for high integrated luminosities ($> 300\text{fb}^{-1}$).

2. **Five lepton final states**: These final states can be produced in several decay channels

$$\begin{aligned} &E^+ N \to \ell^+ Z \ell^\pm W^\mp, \qquad Z \to \ell^+ \ell^-, \quad W \to \ell \nu \\ &E^+ N \to \ell^+ Z \nu Z, \qquad Z \to \ell^+ \ell^- \end{aligned} \tag{5.60}$$

and likewise for $E^- N$. Small additional contributions from W, Z decay to τ leptons are also present. Also, if a lepton in the previous channel is missed by the detector, then a five lepton final state is produced. This is a general feature and may be quoted as

"A final state with a given number of charged leptons will contribute to final states with a lower lepton multiplicity when one of more of the leptons are missed by the detector."

Five lepton signals have larger branching ratios than six lepton signals and also have small backgrounds. However, like the previous case, this mode can signal fermion triplet production only for large integrated luminosities.

3. **$\ell^\pm \ell^\pm \ell^\pm \ell^\mp$ final state**: The $E^\pm N$ production process with decay

$$E^\pm N \to \ell^+ Z \ell^+ W^-, \qquad E^- N \to \ell^- Z \ell^+ W^- \tag{5.61}$$

and similarly for $E^- N$ gives rise to this interesting state with three like-sign leptons and one of opposite sign. Since this cannot be produced in type I and II see-saw mechanisms, so it constitutes a characteristic signature for fermion triplet production. A small but additional contribution arises from $N \to \ell^- W^+$ when ℓ^- is missed by the detector and W^+ decays leptonically. Also, one or two missed leptons from the previous two channels can also contribute to this final state.

4. **$\ell^+ \ell^+ \ell^- \ell^-$ final state**: This is a common channel to both scalar and fermion triplet production. In this case, four leptons can result from many decay channels

$$\begin{aligned}
&E^+ E^- \to \ell^+ Z \ell^- Z, \quad ZZ \to \ell^+ \ell^- q\bar{q}, \ell^+ \ell^- \nu\bar{\nu} \\
&E^+ E^- \to \ell^+ Z \ell^- h / \ell^+ h \ell^- Z, \quad Z \to \ell^+ \ell^-, \quad h \to q\bar{q} \\
&E^+ E^- \to \nu W^+ \ell^- Z / \ell^+ Z \nu W^-, \quad Z \to \ell^+ \ell^-, \quad W \to \ell\nu \\
&E^\pm N \to \ell^\pm Z \ell^- W^+, \quad Z \to \ell^+ \ell^-, \quad W \to q\bar{q}'
\end{aligned} \tag{5.62}$$

Additional contributions come from channels with τ leptons or more charged leptons that are missed. This final state is crucial in establishing the production of the heavy charged lepton E, which appears as a sharp peak in a trilepton invariant mass distribution. Due to the common existence in scalar and fermion triplet production, it is essential to distinguish between the two. Simulation studies [39] show that the like-sign dilepton invariant mass distribution in this case is much broader and has completely different peaks than the one found for scalar triplet case, even for the same pre-selection and selection criteria. Hence, it is possible to distinguish between the two.

5. **$\ell^\pm \ell^\pm \ell^\pm$ final state**: This is a noticeably distinct final state intrinsic to fermion triplet production channels. It is produced when one or several charged leptons are missed by the detector, or from $Z \to \tau^+ \tau^-$ decay when one of the τ leptons decay hadronically and the other one leptonically. The signal is however small like

its SM backgrounds.

6. **$\ell^\pm\ell^\pm\ell^\mp$ final state**: This is once again an excellent final state for the discovery of fermion triplets. Despite having lower lepton multiplicity, the signal is quite clean and has a much larger cross-section compared to channels with larger lepton multiplicity. An added advantage of this channel is that it establishes the production of the heavy neutrino N, which appears as a peak in the invariant mass distribution of two oppositely charged leptons plus the missing momentum. As expected, several decay channels have contributions towards this final state.

$$\begin{aligned}
&E^+N \to \ell^+ Z \ell^\pm W^\mp, \quad Z \to q\bar{q}/\nu\bar{\nu}, \quad W^\pm \to \ell^\pm \nu \\
&E^+N \to \ell^+ h \ell^\pm W^\mp, \quad h \to q\bar{q}, \quad W \to \ell\nu \\
&E^+N \to \bar{\nu} W^+ \ell^\pm W^\mp, \quad W \to \ell\nu
\end{aligned} \tag{5.63}$$

Like every other channel, this also receives contributions from the previously mentioned channels with one or several missing charged leptons. To distinguish between scalar and fermion triplet signals, the discriminant is once again the like-sign dilepton invariant mass distribution. At the level of fast simulations, it is seen that the fermion triplet distribution is broader and has a longer tail relative to the distribution for scalar triplet production. These features indicating an excess of $\ell^\pm\ell^\pm\ell^\mp$ events can be used to identify E^+E^- and/or $E^\pm N$ production.

7. **$\ell^\pm\ell^\pm$ final state**: Decay channels giving like-sign dileptons are

$$\begin{aligned}
&E^+N \to \ell^+ Z \ell^+ W^-, \quad Z \to q\bar{q}/\nu\bar{\nu}, \quad W \to q\bar{q}' \\
&E^+N \to \ell^+ h \ell^+ W^-, \quad h \to q\bar{q}, \quad W \to q\bar{q}' \\
&E^+N \to \nu W^+ \ell^+ W^-, \quad W^+ \to \ell\nu, \quad W^- \to q\bar{q}'
\end{aligned} \tag{5.64}$$

and similarly for E^-N decays. There are obviously contributions from previous channels with missing charged leptons. It was mentioned in the discussion on like-sign dilepton signals for scalar triplets that the invariant mass distribution shows a characteristic sharp peak at $M_{H^{\pm\pm}}$. In contrast, the distribution in this case is broader and has a long tail once again indicating an excess of events. Thus, a distinction can be made between the two scenarios.

8. **$\ell^+\ell^- jjjj$ final state**: Some of the decay modes that can produce opposite sign charged lepton pairs are

$$\begin{aligned}
E^+N &\to \ell^+ Z \ell^- W^+, \quad Z \to q\overline{q}/\nu\overline{\nu}, \quad W \to q\overline{q}' \\
E^+N &\to \ell^+ h \ell^- W^+, \quad h \to q\overline{q}, \quad W \to q\overline{q}' \\
E^+N &\to \overline{\nu} W^+ \ell^- W^-, \quad WW \to \ell\nu q\overline{q}' \\
E^+E^- &\to \ell^+ Z \ell^- Z, \quad Z \to q\overline{q}/\nu\overline{\nu} \\
E^+E^- &\to \ell^+ Z \ell^- h/\ell^+ h \ell^- Z, \quad Z \to q\overline{q}/\nu\overline{\nu}, \quad h \to q\overline{q} \\
E^+E^- &\to \ell^+ h \ell^- h, \quad h \to q\overline{q}
\end{aligned} \tag{5.65}$$

Due to presence of large SM backgrounds, these states are observationally difficult. An inclusive search for these signals is hopeless since they are produced at an invariant mass which is much lower than to separate them from the background. In case of some specific models like those involving Z' bosons, however, the decay $Z' \to \ell^+\ell^-$ produces these dileptons at an invariant mass $M_{\ell_1\ell_2} \simeq M_{Z'}$ with an enhancement in cross-section from the on-shell Z' propagator (resonant production). But, the approach in this entire analysis is conservative and hence no such enhancement is taken into account. Thus, we concentrate on final states with four jets.

9. **$\ell^\pm jjjj$ final state**: Decays leading to the production of only one charged lepton are

$$\begin{aligned}
E^+N &\to \overline{\nu} W^+ \ell^\mp W^\pm, \quad W \to q\overline{q}' \\
E^+N &\to \ell^+ Z \nu h/\ell^+ \nu h h/\ell^+ \nu Z Z, \quad Z \to q\overline{q}'/\nu\overline{\nu}, \quad h \to q\overline{q} \\
E^+E^- &\to \ell^+ Z \nu W^-/\ell^+ \nu h W^-, \quad Z \to q\overline{q}/\nu\overline{\nu}, \quad h \to q\overline{q}, \quad W \to q\overline{q}' \\
E^+E^- &\to \overline{\nu} W^+ \ell^- Z/\overline{\nu} W^+ \ell^- h, \quad Z \to q\overline{q}/\nu\overline{\nu}, \quad h \to q\overline{q}, \quad W \to q\overline{q}'
\end{aligned} \tag{5.66}$$

Although they have a high branching ratio, they suffer from very large SM backgrounds of which the dominant ones are Wnj and $t\bar{t}nj$ in the dilepton decay channel, with one W boson decaying into a τ which subsequently decays hadronically, or maybe an electron/muon which are missed by the detector. As such, observation of these signals are very difficult.

5.15 Viewpoint

- Fermion triplet production at the LHC can lead to a large number of possible final state signatures involving charged leptons.

- Some of these final states are unique to the type III scenario while rest are common to types I and/or II see-saw. Unique to the heavy fermion triplet picture are the final states involving three like-sign leptons, five and six charged leptons.
- The six lepton final state has a very small branching ratio and requires very high luminosities and hence not promising for early discoveries at the LHC.
- Final states with three like-sign leptons and five leptons offer possibilities of observation at much lower luminosities. The four lepton signals is further interesting as it exhibits observation potential at almost half the luminosity of the former final states.
- The $\ell^\pm\ell^\pm\ell^\mp$ final state is however, the best possible channel showing discovery potential for $m_E \sim 300$ GeV and at less than 3 fb^{-1}. It also provides evidence for heavy neutrino, N production.
- The like-sign dilepton channel shows almost similar discovery potential as the $\ell^\pm\ell^\pm\ell^\mp$ channel. The fermion triplet signal can be distinguished from the scalar triplet signal by examining the invariant mass distribution patterns.
- The final states $\ell^+\ell^- jjjj$ and $\ell^\pm jjjj$ are very difficult to observe due to large SM backgrounds.

5.16 Final hope

The see-saw mechanism is a simple, yet very elegant way of explaining the puzzling observation of the huge mass disparity of neutrinos compared to other SM leptons. In this chapter, I reviewed the possibility of observation of see-saw signals at the LHC through multi-lepton signatures, which is convenient in terms of experimental searches. In the words of Richard Feynman

"It doesn't matter how beautiful your theory is, it doesn't matter how smart you are; if it doesn't agree with experiment, it's wrong"

This statement is in a way highly applicable as well as motivating in the current premise of validating the theory of see-saw through experimental data. Although everything that is discussed in this review are based on optimistic speculations and results of fast simulation studies, it is this optimism that drives physicists, both in theory and experiment to go beyond the limits and search for any truth that might be lying yet undiscovered around us. The LHC at CERN has certainly provided further boost to this optimism by providing evidence for the Higgs boson earlier in 2012. In the coming years, it is anticipated and would be indeed beneficial to neutrino physics and the entire physics community as a whole if the LHC can detect indirect but positive signals for either of the see-saw mechanisms. Till then, we can only keep our ambitions high and move ahead undeterred in this quest for truth.

Chapter 6

The type III see-saw mechanism with two Higgs doublets

In this chapter, I delineate the theoretical framework of the type III see-saw mechanism with two Higgs-doublets . Such a model offers a richer phenomenology than the ordinary type III picture. An extra discrete Z_2 symmetry is imposed and the complete theory is worked out in the CP-conserving case. Section 6.2 describes the triplet interactions with the scalars. Similar to the case mentioned in section 5.14 of chapter 5, the possible final states for E^+E^- and $E^\pm N$ production channels are tabulated in Tables 6 and 7.

6.1 The 2HDM

The 2HDM [43] is the simplest extension of the Standard Model in which the scalar sector is augmented by an $SU(2)_L$ doublet having identical quantum numbers as the Standard Model Higgs doublet (I=1/2, Y=+1). In this section, I begin with the most general, renormalizable and gauge-invariant Lagrangian. Then I discuss the motivation for simplifying the picture by imposing a Z_2 symmetry and find the subsequent conditions on the 2HDM parameters. The physical spectrum is worked out alongwith the gauge interactions of the scalars.

6.1.1 The Lagrangian

The most general Higgs Lagrangian for the 2HDM with two Higgs doublets Φ_1 and Φ_2 having I=1/2 and Y=+1 is given by [44]

$$\mathcal{L} = T - V \tag{6.1}$$

$$
\begin{aligned}
T &= (\mathcal{D}^\mu\Phi_1)^\dagger(\mathcal{D}_\mu\Phi_1) + (\mathcal{D}^\mu\Phi_2)^\dagger(\mathcal{D}_\mu\Phi_2) + \xi(\mathcal{D}^\mu\Phi_1)^\dagger(\mathcal{D}_\mu\Phi_2) + \xi^*(\mathcal{D}^\mu\Phi_2)^\dagger(\mathcal{D}_\mu\Phi_1) \\
V &= -\frac{1}{2}\{m_{11}^2(\Phi_1^\dagger\Phi_1) + m_{22}^2(\Phi_2^\dagger\Phi_2) + [m_{12}^2(\Phi_1^\dagger\Phi_2) + h.c]\} \\
&+ \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
&+ \frac{1}{2}[\lambda_5(\Phi_1^\dagger\Phi_2)^2 + h.c] + \{[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)](\Phi_1^\dagger\Phi_2) + h.c\}
\end{aligned}
\tag{6.2}
$$

By hermiticity of the potential, the parameters m_{11}^2, m_{22}^2, λ_{1-4} are real whereas m_{12}^2, λ_{5-7} and ξ can in general be complex. So, in this general framework, there are 14 independent parameters in the Higgs potential and 16 in total for the entire Lagrangian.

6.1.2 Motivation for Z_2 symmetry

The 2HDM, although offering the possibility of a richer phenomenology, has a severe problem of having FCNC's, mediated by neutral scalars. As an example, the general form of the Yukawa interaction term involving d-type quarks (Q=–1/3) and 2HDM scalars will be

$$
\mathcal{L}^D_{Yukawa} = -(\overline{Q'_L}Y_1'^D\Phi_1 q_R'^D + \overline{Q'_L}Y_2'^D\Phi_2 q_R'^D) + h.c \tag{6.3}
$$

The mass matrix is given by

$$
M = Y_1'^D\frac{v_1}{\sqrt{2}} + Y_2'^D\frac{v_2}{\sqrt{2}} \tag{6.4}
$$

In the Standard Model, diagonalizing the mass matrix automatically diagonalizes the Yukawa interactions thus eliminating the FCNC's. In this case, however, $Y_1'^D$ and $Y_2'^D$ cannot in general be diagonalized simultaneously and hence the Yukawa interactions are not flavor diagonal. Neutral scalars will mediate FCNC's. These "*dangerous*" FCNC's have the potential of bringing about several changes in the phenomenology. By imposing a discrete/continuous symmetry, these tree-level FCNC's can be prevented.

In the present scenario, we impose a Z_2 symmetry to naturally suppress the FCNC's. Under this symmetry operation, the Lagrangian is invariant under the interchange

$$
\Phi_1 \leftrightarrow \Phi_1, \quad \Phi_2 \leftrightarrow -\Phi_2 \quad \text{or} \quad \Phi_1 \leftrightarrow -\Phi_1, \quad \Phi_2 \leftrightarrow \Phi_2 \tag{6.5}
$$

In short, $\Phi_1 \leftrightarrow \Phi_2$ transitions are forbidden.

With the Z_2-charge assignments of +1 and –1 to Φ_1 and Φ_2 respectively, there arises four types of models depending on four independent Z_2-charge assignments to the Standard Model leptons and quarks.

	Φ_1	Φ_2	q_R^U	q_R^D	ℓ_R	Q_L, L_L
Type I	+	−	−	−	−	+
Type II	+	−	−	+	+	+
Type X	+	−	−	−	+	+
Type Y	+	−	−	+	−	+

Table 5: Charge assignments of the Z_2 symmetry.

I will work with the type II 2HDM and assign Z_2 charge of –1 to the heavy fermion triplet Σ_L so that it couples only to the Φ_2 scalar doublet. In the type II 2HDM, the Standard Model leptons, u-type and d-type quarks have Yukawa interactions with Φ_1, Φ_2 and Φ_1 respectively.

It is evident from the general form of the Lagrangian given by (6.1) and (6.2) that it violates Z_2 symmetry thus allowing $\Phi_1 \leftrightarrow \Phi_2$ transitions. The dim-2 operators proportional to m_{12}^2 lead to a *soft* violation of Z_2 symmetry and the dim-4 operators proportional to $\lambda_{6,7}$, ξ lead to a *hard* violation of Z_2 symmetry.

An exact Z_2 symmetry is ensured by imposing $\lambda_6 = \lambda_7 = m_{12}^2 = \xi = 0$, thus allowing only one parameter λ_5 to be complex. In case of soft violation, m_{12}^2 and λ_5 can be complex. I consider the latter case in the coming discussion. The advantage of working in the soft Z_2 violating picture will be discussed after I derive the physical particle spectrum in subsection 6.1.4. Moreover, the CP conserved picture is taken and hence all the parameters in the potential are assumed to be real.

The final form of the Lagrangian which I will be working with this becomes

$$\mathcal{L} = T - V$$

where

$$\begin{aligned}T &= (\mathcal{D}^\mu\Phi_1)^\dagger(\mathcal{D}_\mu\Phi_1) + (\mathcal{D}^\mu\Phi_2)^\dagger(\mathcal{D}_\mu\Phi_2)\\ V &= -\frac{1}{2}\{m_{11}^2(\Phi_1^\dagger\Phi_1) + m_{22}^2(\Phi_2^\dagger\Phi_2) + m_{12}^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1)\}\\ &+ \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\\ &+ \frac{\lambda_5}{2}\{(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2\}\end{aligned} \tag{6.6}$$

The exact Z_2 symmetry situation is realized for $m_{12}^2 = 0$.

6.1.3 Vacuum expectation values

The most general vacuum state can be described by

$$\langle\Phi_1^0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_1\end{pmatrix}, \qquad \langle\Phi_2^0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}u\\ v_2e^{i\chi}\end{pmatrix} \tag{6.7}$$

with v_1,v_2,$\chi \in \mathbb{R}$. For vacuum with $u \neq 0$, the electric charge is not conserved and photon becomes massive. Under $u = 0$, neutral vacua are possible. To preserve CP symmetry, I work with vacua having real VEV's.

$$\langle\Phi_1^0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_1\end{pmatrix}, \qquad \langle\Phi_2^0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_2\end{pmatrix} \tag{6.8}$$

The minimization conditions are

$$\left.\frac{\partial V}{\partial\Phi_1}\right|_{\substack{\Phi_1=\langle\Phi_1^0\rangle,\\ \Phi_2=\langle\Phi_2^0\rangle}} = 0, \qquad \left.\frac{\partial V}{\partial\Phi_2}\right|_{\substack{\Phi_1=\langle\Phi_1^0\rangle,\\ \Phi_2=\langle\Phi_2^0\rangle}} = 0. \tag{6.9}$$

which reduce to

$$\lambda_1 v_1^2 + (\lambda_3 + \lambda_4 + \lambda_5)v_2^2 = m_{12}^2\frac{v_2}{v_1} + m_{11}^2 \tag{6.10}$$

$$\lambda_2 v_2^2 + (\lambda_3 + \lambda_4 + \lambda_5)v_1^2 = m_{12}^2\frac{v_1}{v_2} + m_{22}^2 \tag{6.11}$$

For exact Z_2 symmetry, these further reduce to

$$\lambda_1 v_1^2 + (\lambda_3 + \lambda_4 + \lambda_5)v_2^2 = m_{11}^2 \tag{6.12}$$

$$\lambda_2 v_2^2 + (\lambda_3 + \lambda_4 + \lambda_5)v_1^2 = m_{22}^2 \tag{6.13}$$

6.1.4 The physical Higgs sector

Writing the scalar doublets in terms of field components

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{v_1+\rho_1+i\eta_1}{\sqrt{2}} \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{v_2+\rho_2+i\eta_2}{\sqrt{2}} \end{pmatrix} \tag{6.14}$$

the following particle spectrum is obtained by decomposing the potential given by eqn. (6.6) and extracting mass terms.

1. Charged scalars: The mass term for the charged scalars is given by

$$\mathcal{L}_{mass,+} = \frac{1}{2}[m_{12}^2 - (\lambda_4+\lambda_5)v_1v_2] \begin{pmatrix} \phi_1^- & \phi_2^- \end{pmatrix} \begin{pmatrix} v_2/v_1 & -1 \\ -1 & v_1/v_2 \end{pmatrix} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix} \tag{6.15}$$

The squared mass matrix is given by

$$\mathcal{M}_\pm^2 = \frac{1}{2}[m_{12}^2 - (\lambda_4+\lambda_5)v_1v_2] \begin{pmatrix} v_2/v_1 & -1 \\ -1 & v_1/v_2 \end{pmatrix} \tag{6.16}$$

The mass eigenvalues are given by

$$m_{G^\pm}^2 = 0, \quad m_{H^\pm}^2 = \frac{1}{2}(v_1^2+v_2^2)\left[\frac{m_{12}^2}{v_1v_2} - (\lambda_4+\lambda_5)\right] \tag{6.17}$$

The physical fields $G^\pm$ and $H^\pm$ are given by

$$\begin{aligned} G^\pm &= \cos\beta\phi_1^\pm + \sin\beta\phi_2^\pm \qquad \text{(Goldstone boson)} \\ H^\pm &= -\sin\beta\phi_1^\pm + \cos\beta\phi_2^\pm \qquad \text{(physical Higgs boson)} \end{aligned} \tag{6.18}$$

where

$$\begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \mathcal{M}_\pm^2 \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & m_{H^\pm}^2 \end{pmatrix} \tag{6.19}$$

The diagonalization conditions lead to

$$\begin{aligned} \sin\beta\cos\beta &= \frac{v_1v_2}{v_1^2+v_2^2} \\ \sin^2\beta &= \frac{v_2^2}{v_1^2+v_2^2} \\ \cos^2\beta &= \frac{v_1^2}{v_1^2+v_2^2} \end{aligned} \tag{6.20}$$

Since v_1 and v_2 are taken to be non-negative, hence $\sin\beta\cos\beta$ is positive. However, $\sin\beta$ or $\cos\beta$ individually have a sign ambiguity. A better parameter is $\tan\beta =$

v_2/v_1 which is positive. β can be assumed to be in the first quadrant without loss of generality [45]. With this convention

$$\sin\beta = \frac{v_2}{\sqrt{v_1^2+v_2^2}}, \quad \cos\beta = \frac{v_1}{\sqrt{v_1^2+v_2^2}} \tag{6.21}$$

2. CP-even neutral Higgs: The mass term is given by

$$\mathcal{L}^{even}_{mass,0} = \frac{1}{2}\begin{pmatrix}\rho_1 & \rho_2\end{pmatrix}\begin{pmatrix}A & C\\ C & B\end{pmatrix}\begin{pmatrix}\rho_1\\ \rho_2\end{pmatrix} \tag{6.22}$$

with the squared mass matrix as

$$\mathcal{M}^2_{CP_{even}} = \begin{pmatrix}A & C\\ C & B\end{pmatrix} \tag{6.23}$$

where

$$\begin{aligned} A &= \frac{m_{12}^2}{2}\frac{v_2}{v_1} + \lambda_1 v_1^2 \\ B &= \frac{m_{12}^2}{2}\frac{v_1}{v_2} + \lambda_2 v_2^2 \\ C &= -\frac{m_{12}^2}{2} + (\lambda_3+\lambda_4+\lambda_5)v_1 v_2 \end{aligned} \tag{6.24}$$

The mass eigenvalues are given by

$$m^2_{h^0} = \frac{1}{2}\left[(A+B) - \sqrt{(A-B)^2+4C^2}\right] \tag{6.25}$$

$$m^2_{H^0} = \frac{1}{2}\left[(A+B) + \sqrt{(A-B)^2+4C^2}\right] \tag{6.26}$$

Here h^0 and H^0 are the light and heavy neutral Higgses respectively. These physical fields are written as

$$\begin{aligned} h^0 &= \cos\alpha\rho_1 + \sin\alpha\rho_2 \\ H^0 &= -\sin\alpha\rho_1 + \cos\alpha\rho_2 \end{aligned} \tag{6.27}$$

where

$$\begin{pmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}A & C\\ C & B\end{pmatrix}\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix} = \begin{pmatrix}m^2_{h^0} & 0\\ 0 & m^2_{H^0}\end{pmatrix} \tag{6.28}$$

The diagonalization conditions in this case lead to

$$\begin{aligned} A &= m_{h^0}^2 \cos^2\alpha + m_{H^0}^2 \sin^2\alpha \\ B &= m_{h^0}^2 \sin^2\alpha + m_{H^0}^2 \cos^2\alpha \\ C &= \sin\alpha\cos\alpha(m_{h^0}^2 - m_{H^0}^2) \end{aligned} \tag{6.29}$$

so that

$$\tan 2\alpha = \frac{2C}{A-B} \tag{6.30}$$

3. CP odd neutral Higgs: The mass term is given by

$$\mathcal{L}^{odd}_{mass,0} = \frac{1}{2}\begin{pmatrix}\eta_1 & \eta_2\end{pmatrix} \mathcal{M}^2_{CP_{odd}} \begin{pmatrix}\eta_1 \\ \eta_2\end{pmatrix} \tag{6.31}$$

where

$$\mathcal{M}^2_{CP_{odd}} = \left(\frac{m_{12}^2}{2} - \lambda_5 v_1 v_2\right)\begin{pmatrix} v_2/v_1 & -1 \\ -1 & v_1/v_2 \end{pmatrix} \tag{6.32}$$

is the squared mass matrix. The eigenvalues are given by

$$m_{G^0}^2 = 0, \quad m_{A^0}^2 = (v_1^2+v_2^2)\left(\frac{m_{12}^2}{2v_1v_2} - \lambda_5\right) \tag{6.33}$$

The physical fields are given by

$$\begin{aligned} G^0 &= \cos\gamma\eta_1 + \sin\gamma\eta_2 \\ A^0 &= -\sin\gamma\eta_1 + \cos\gamma\eta_2 \end{aligned} \tag{6.34}$$

The diagonalization conditions lead to

$$\begin{aligned} \sin\gamma\cos\gamma &= \frac{v_1v_2}{v_1^2+v_2^2} \\ \frac{v_1}{v_2}\sin^2\gamma &= \frac{v_2}{v_1}\cos^2\gamma \end{aligned} \tag{6.35}$$

Thus, with the chosen convention and comparing relations (6.20) and (6.35), we have $\gamma = \beta$.

To summarize, there are five physical Higgs bosons h^0 and H^0 (neutral scalars), H^+ and H^- (singly charged scalars), A^0 (neutral pseudoscalar). The three would-be Goldstone bosons G^0, G^+ and G^- would be 'eaten' by the three gauge bosons W^+, W^- and Z thereby becoming massive.

6.1.5 The kinetic term and gauge interactions

The kinetic term is given by

$$\mathcal{L}_{kinetic} = (\mathcal{D}^\mu \Phi_1)^\dagger (\mathcal{D}_\mu \Phi_1) + (\mathcal{D}^\mu \Phi_2)^\dagger (\mathcal{D}_\mu \Phi_2) \tag{6.36}$$

The interactions of the physical Higgses with the gauge bosons can be obtained by simplifying the above equation using the expressions obtained in subsection 6.1.4. It is obvious that the range of interactions are much larger in this picture due to the wider spectrum of particles. For simplicity in comprehending the interactions, I divide them into three categories

- Interactions with $W^\pm$ bosons
- Interactions with Z bosons
- Interactions with the electromagnetic field (photons).

1. Interactions with $W^\pm$ bosons: The interaction term is given by

$$\begin{aligned}\mathcal{L}_W =& \frac{g^2 v_1}{2\cos\beta}\cos(\alpha-\beta) W^{-\mu} W^+_\mu h^0 - \frac{g^2 v_1}{2\cos\beta}\sin(\alpha-\beta) W^{-\mu} W^+_\mu H^0 \\ &+ \frac{g^2}{2} W^{-\mu} W^+_\mu H^+ H^- + \frac{g^2}{4} W^{-\mu} W^+_\mu (h^0)^2 + \frac{g^2}{4} W^{-\mu} W^+_\mu (H^0)^2 + \frac{g^2}{4} W^{-\mu} W^+_\mu (A^0)^2 \\ &+ \frac{g}{2}\left[\{(\partial_\mu A^0) H^- - (\partial_\mu H^-) A^0\} W^{+\mu} + \{(\partial_\mu A^0) H^+ - (\partial_\mu H^+) A^0\} W^{-\mu}\right] \\ &- \frac{ig}{2}\sin(\alpha-\beta)\left[\{(\partial_\mu h^0) H^- - (\partial_\mu H^-) h^0\} W^{+\mu} - \{(\partial_\mu h^0) H^+ - (\partial_\mu H^+) h^0\} W^{-\mu}\right] \\ &- \frac{ig}{2}\cos(\alpha-\beta)\left[\{(\partial_\mu H^0) H^- - (\partial_\mu H^-) H^0\} W^{+\mu} - \{(\partial_\mu H^0) H^+ - (\partial_\mu H^+) H^0\} W^{-\mu}\right]\end{aligned} \tag{6.37}$$

   The terms in the second line show that the interactions are Standard Model-like. As will be observed, the mixing angle parameters $\sin(\alpha-\beta)$, $\cos(\alpha-\beta)$ and $\tan\beta$ are the important ones as they discern the interactions of this model from the Standard Model ones.

2. Interactions with Z bosons: The interaction term is given by

$$
\begin{aligned}
\mathcal{L}_Z =& \frac{g^2 v_1}{4\cos^2\theta_W \cos\beta}\cos(\alpha-\beta) Z^\mu Z_\mu h^0 - \frac{g^2 v_1}{4\cos^2\theta_W\cos\beta}\sin(\alpha-\beta) Z^\mu Z_\mu H^0 \\
&+ \frac{g^2}{8\cos^2\theta_W} Z^\mu Z_\mu (h^0)^2 + \frac{g^2}{8\cos^2\theta_W} Z^\mu Z_\mu (H^0)^2 + \frac{g^2}{8\cos^2\theta_W} Z^\mu Z_\mu (A^0)^2 \\
&+ \frac{g^2}{4\cos^2\theta_W}(1-2\sin^2\theta_W)^2 Z^\mu Z_\mu H^- H^+ \\
&+ \frac{g}{2\cos\theta_W}\sin(\alpha-\beta)\left[(\partial_\mu h^0)A^0 - (\partial_\mu A^0)h^0\right] Z^\mu \\
&+ \frac{g}{2\cos\theta_W}\sin(\alpha+\beta)\left[(\partial_\mu A^0)H^0 - (\partial_\mu H^0)A^0\right] Z^\mu \\
&+ \frac{ig}{2\cos\theta_W}(1-2\sin^2\theta_W)\left[(\partial_\mu H^-)H^+ - (\partial_\mu H^+)H^-\right] Z^\mu
\end{aligned}
\tag{6.38}
$$

3. Interactions with photons: The interaction term is given by

$$
\begin{aligned}
\mathcal{L}_\gamma =& ie\left[(\partial_\mu H^-)H^+ - (\partial_\mu H^+)H^-\right] A^\mu + e^2 A^\mu A_\mu H^- H^+ \\
&+ \frac{eg}{\cos\theta_W}(1-2\sin^2\theta_W) A^\mu Z_\mu H^- H^+
\end{aligned}
\tag{6.39}
$$

The masses of the W and Z bosons are given by

$$
\begin{aligned}
M_W^2 &= \frac{g^2(v_1^2+v_2^2)}{4} \\
M_Z^2 &= \frac{g^2(v_1^2+v_2^2)}{4\cos^2\theta_W}
\end{aligned}
\tag{6.40}
$$

Thus, for the correct electroweak scale $\sqrt{v_1^2+v_2^2} = v = 246$ GeV. The ρ parameter is equal to unity which is expected because of the basic construction of the doublet structure which preserves custodial symmetry at the tree level.

6.1.6 Decoupling of heavy Higgs bosons

In several analyses of 2HDM, a Standard Model-like picture is assumed with the light CP-even neutral Higgs h^0 being similar to the Higgs boson of the Standard Model, whereas the other Higgs bosons are heavier and hence escape observation. Often, these heavy masses are assumed to be of the order of Λ or the scale of New Physics and a *decoupling* [44,46-48] property is assigned which means that

"*the correct description of the observable phenomenon must be valid for the (unphysical) limit* $M \to \infty$"

Quite interestingly, the 2HDM offers the possibility of a different kind of realization of the above mentioned scenario. If we take a look at the formulae for the masses of physical particles obtained in subsection 6.1.4, it is evident that large masses of Higgs particles are viable for large values of the parameters $\nu = m_{12}^2/2v_1v_2$ and/or λ_i's. The quartic terms λ_i's of Higgs couplings are transformed to the quartic self-couplings of the physical Higgses. For perturbative calculations to be trustworthy, the magnitudes of these couplings must not be too large. However, there is no such restriction on ν. The theoretical constraints on the parameters in the potential are mentioned in Appendix C.

In 2HDM, the decoupling limit translates to

$$\nu >> \mid \lambda_i \mid \qquad (i = 1, 2, 3, 4, 5) \tag{6.41}$$

The effect on the particle spectrum is as follows

1. CP-even Higgses: A little bit of algebra shows that the terms A, B and C arising in the mass formulae for h^0 and H^0 can be written in terms of ν and β as

$$\begin{aligned} A &= \nu v^2 \sin^2\beta + \lambda_1 v^2 \cos^2\beta \\ B &= \nu v^2 \cos^2\beta + \lambda_2 v^2 \sin^2\beta \\ C &= -\frac{\nu v^2 \sin 2\beta}{2}\left[1 - \frac{(\lambda_3 + \lambda_4 + \lambda_5)}{\nu}\right] \end{aligned} \tag{6.42}$$

so that to leading order in $\mid \lambda_i \mid /\nu$, the squared masses are

$$\begin{aligned} \frac{m_{h^0}^2}{v^2} &\simeq \lambda_1 \cos^4\beta + \lambda_2 \sin^4\beta + 2\sin^2\beta\cos^2\beta(\lambda_3 + \lambda_4 + \lambda_5) \\ \frac{m_{H^0}^2}{v^2} &\simeq \nu + \sin^2\beta\cos^2\beta\left[\lambda_1 + \lambda_2 - 2(\lambda_3 + \lambda_4 + \lambda_5)\right] \end{aligned} \tag{6.43}$$

Thus, in the decoupling limit, we have $m_{H^0}^2 \simeq \nu v^2$ and a light h^0 due to small values of λ_i's.

2. CP-odd Higgs: In terms of ν, the squared mass of A^0 is

$$m_{A^0}^2 = \nu v^2 \left(1 - \frac{\lambda_5}{\nu}\right) \tag{6.44}$$

so that in the decoupling limit, $m_{A^0}^2 \simeq \nu v^2$.

3. Charged Higgses: The squared masses of the charged Higgses are given by

$$m_{H^\pm}^2 = \nu v^2 \left[1 - \frac{(\lambda_4 + \lambda_5)}{2\nu} \right] \tag{6.45}$$

which in the decoupling limit reduces to $m_{H^\pm}^2 \simeq \nu v^2$.

Also, the mixing angles α and β are nearly equal in this limit. From eqns. (6.42), we get

$$A - B = \nu v^2 \left[-\cos 2\beta + (\frac{\lambda_1}{\nu} \cos^2\beta - \frac{\lambda_2}{\nu} \sin^2\beta) \right] \tag{6.46}$$

and using eqn. (6.30), we get in the decoupling limit $\tan 2\alpha = \tan 2\beta$. In our discussion, $\alpha \in (0, \frac{\pi}{2})$ and β is in the first quadrant which means $\alpha = \beta$. Moreover, from eqns. (6.37) and (6.38) it can be seen that only the light Higgs h^0 couple to the Standard Model gauge bosons W and Z and the interactions are also Standard Model-like.

To summarize, in the decoupling limit the light CP-even Higgs h^0 behave like the Standard Model Higgs and the other Higgses are very heavy and almost degenerate in mass. This is a significant advantage of working with the soft Z_2 violating picture rather than an exact Z_2 symmetry since decoupling limit is not possible in the latter case.

6.2 Triplet interactions with type II 2HDM

Assigning a Z_2 charge of –1 to the heavy fermionic triplet Σ_L, its Yukawa interactions are restricted only with the scalar doublet Φ_2.

The triplet Lagrangian is given by

$$\begin{aligned} \mathcal{L}_\Delta = & Tr[\overline{\Sigma_L} i \not{D} \Sigma_L] - \frac{1}{2} Tr[\overline{\Sigma_L^C} M_\Sigma \Sigma_L + \overline{\Sigma_L} M_\Sigma \Sigma_L^C] \\ & - \left(\overline{L_L} \sqrt{2} Y_{2\Sigma}^\dagger \Sigma_L^C \widetilde{\Phi_2} + \widetilde{\Phi_2}^\dagger \overline{\Sigma_L^C} \sqrt{2} Y_{2\Sigma} L_L \right) \end{aligned} \tag{6.47}$$

where $Y_{2\Sigma}$ denotes the Yukawa coupling with the scalar doublet Φ_2. The Yukawa interaction of the Standard Model lepton doublets is given by

$$\mathcal{L}_{Y,SM} = - \left(\overline{L_L} Y_{1\ell}^\dagger \Phi_1 \ell_R + \overline{\ell_R} \Phi_1^\dagger Y_{1\ell} L_L \right) \tag{6.48}$$

The charged and neutral lepton mass terms in this case are given by

$$\begin{aligned}\mathcal{L}_{CLepton} &= -\Big[\begin{pmatrix}\overline{\boldsymbol{\ell}_R}' & \overline{E_R'}\end{pmatrix}\begin{pmatrix}\frac{Y_{1\ell}v_1}{\sqrt{2}} & 0\\ Y_{2\Sigma}v_2 & M_\Sigma\end{pmatrix}\begin{pmatrix}\boldsymbol{\ell}_L'\\ E_L'\end{pmatrix} + h.c\Big]\\ \mathcal{L}_{NLepton} &= -\frac{1}{2}\Big[\begin{pmatrix}\overline{\boldsymbol{\nu}'^C}_L & \overline{N'}_R\end{pmatrix}\begin{pmatrix}0 & \frac{Y_{2\Sigma}^T v_2}{\sqrt{2}}\\ \frac{Y_{2\Sigma}v_2}{\sqrt{2}} & M_\Sigma\end{pmatrix}\begin{pmatrix}\boldsymbol{\nu}'_L\\ N'_L\end{pmatrix} + h.c\Big]\end{aligned} \tag{6.49}$$

These matrices are diagonalized in a manner similar to the one mentioned in Appendix B. The only replacements that need to be done in the results presented in eqn. (4.29) are $Y_\Sigma \to Y_{2\Sigma}$,$v \to v_2$ and $m_\ell = Y_{1\ell}v_1/\sqrt{2}$.

The mixing between the heavy and light neutrinos is determined by

$$V_{\ell N} \simeq -\frac{v_2 Y_{2\Sigma}^\dagger M_\Sigma^{-1}}{\sqrt{2}} \tag{6.50}$$

and the light neutrino masses are given by

$$m^\nu_{light} \simeq -\frac{v_2^2}{2} Y_{2\Sigma}^T M_\Sigma^{-1} Y_{2\Sigma} \tag{6.51}$$

6.2.1 Gauge Interactions

The gauge interactions involving the heavy charged and neutral leptons with the W and Z bosons are identical to those presented in section 4.3 of chapter 4. I merely rewrite them here for quick reference.

$$\begin{aligned}\mathcal{L}_\gamma &= e(\overline{E}\gamma^\mu E + \overline{\boldsymbol{\ell}}\gamma^\mu \boldsymbol{\ell})A_\mu\\ \mathcal{L}_Z &= g\cos\theta_W \overline{E}\gamma^\mu E Z_\mu + \frac{g}{2\cos\theta_W}\Big[\overline{\boldsymbol{\nu}}(V^\dagger_{PMNS}V_{\ell N}\gamma^\mu P_L = V^T_{PMNS}V^*_{\ell N}\gamma^\mu P_R)N\Big] Z_\mu\\ &+ \frac{g}{\sqrt{2}\cos\theta_W}(\overline{E}V^\dagger_{\ell N}\gamma^\mu P_L\boldsymbol{\ell} + \overline{\boldsymbol{\ell}}V_{\ell N}\gamma^\mu P_L E)Z_\mu\\ \mathcal{L}_W &= -g(\overline{E}\gamma^\mu N W^-_\mu + \overline{N}\gamma^\mu E W^+_\mu) - g(\overline{E}\gamma^\mu V^T_{\ell N}P_R\boldsymbol{\nu}W^-_\mu + \overline{\boldsymbol{\nu}}V^*_{\ell N}\gamma^\mu P_R E W^+_\mu)\\ &- \frac{g}{\sqrt{2}}(\overline{\boldsymbol{\ell}}\gamma^\mu P_L V_{\ell N} N W^-_\mu + \overline{N}\gamma^\mu V^\dagger_{\ell N}P_L\boldsymbol{\nu}W^+_\mu)\end{aligned} \tag{6.52}$$

The only difference lies in the Yukawa interactions. Both the Standard Model lepton-Yukawa interactions and those with the heavy fermionic triplet are modified with respect to the type III see-saw scenario with one Higgs doublet.

1. The Standard Model lepton-Yukawa interactions: In terms of non-eigen components, this is given by

$$\mathcal{L}_{Y,SM} = -\Big[(\phi_1^- \overline{\boldsymbol{\ell}'_R} Y_{1\ell} \boldsymbol{\nu}'_L + \phi_1^{0*} \overline{\boldsymbol{\ell}'_R} Y_{1\ell} \boldsymbol{\ell}'_L) + h.c\Big] \tag{6.53}$$

In terms of mass eigenstates, this is given by

$$\begin{aligned}\mathcal{L}_{Y,SM} = \Big[&(-\sin\beta \overline{\boldsymbol{\ell}} Y_{1\ell} V_{PMNS} P_L \boldsymbol{\nu} H^- + \sin\beta \overline{\boldsymbol{\ell}} Y_{1\ell} V_{\ell N} P_L N + \frac{\cos\alpha}{\sqrt{2}} \overline{\boldsymbol{\ell}} Y_{1\ell} P_L \boldsymbol{\ell} h^0 \\ &- \cos\alpha \overline{\boldsymbol{\ell}} Y_{1\ell} V_{\ell N} P_L E h^0 - \frac{\sin\alpha}{\sqrt{2}} \overline{\boldsymbol{\ell}} Y_{1\ell} P_L \boldsymbol{\ell} H^0 + \sin\alpha \overline{\boldsymbol{\ell}} Y_{1\ell} V_{\ell N} P_L E H^0 \\ &+ \frac{i\sin\beta}{\sqrt{2}} \overline{\boldsymbol{\ell}} Y_{1\ell} P_L \boldsymbol{\ell} A^0 - i\sin\beta \overline{\boldsymbol{\ell}} Y_{1\ell} V_{\ell N} P_L E A^0)\Big] + h.c\end{aligned} \tag{6.54}$$

Using $m_\ell = Y_{1\ell} v_1/\sqrt{2}$, this can also be written as

$$\begin{aligned}\mathcal{L}_{Y,SM} = \Big[&(-\frac{\sqrt{2}}{v_1} \sin\beta \overline{\boldsymbol{\ell}} m_\ell V_{PMNS} P_L \boldsymbol{\nu} H^- + \frac{\sqrt{2}}{v_1} \sin\beta \overline{\boldsymbol{\ell}} m_\ell V_{\ell N} P_L N + \frac{\cos\alpha}{v_1} \overline{\boldsymbol{\ell}} m_\ell P_L \boldsymbol{\ell} h^0 \\ &- \frac{\sqrt{2}}{v_1} \cos\alpha \overline{\boldsymbol{\ell}} m_\ell V_{\ell N} P_L E h^0 - \frac{\sin\alpha}{v_1} \overline{\boldsymbol{\ell}} m_\ell P_L \boldsymbol{\ell} H^0 + \frac{\sqrt{2}}{v_1} \sin\alpha \overline{\boldsymbol{\ell}} m\ell V_{\ell N} P_L E H^0 \\ &+ \frac{i\sin\beta}{v_1} \overline{\boldsymbol{\ell}} m_\ell P_L \boldsymbol{\ell} A^0 - i\frac{\sqrt{2}}{v_1} \sin\beta \overline{\boldsymbol{\ell}} m_\ell V_{\ell N} P_L E A^0)\Big] + h.c\end{aligned} \tag{6.55}$$

Comparing this with the case of only a single Higgs doublet, the number of possible decays is much larger. In the Type III see-saw+1HDM, only possible decay is $E \to \ell h$ whereas the following possibilities are opened up in the type III+2HDM picture

$$N \to H^\mp \ell^\pm, \quad E \to \ell h^0/H^0/A^0 \tag{6.56}$$

2. Triplet-Yukawa interaction: The Yukawa interaction with the heavy fermion triplet in the weak eigenbasis is given by

$$\mathcal{L}_{Y,\Delta} = -\Big[(\phi_2^0 \overline{\Sigma_L^{0C}} Y_{2\Sigma} \boldsymbol{\nu}'_L - \sqrt{2}\phi_2^+ \overline{\Sigma_L^{-C}} Y_{2\Sigma} \boldsymbol{\nu}'_L + \sqrt{2}\phi_2^0 \overline{\Sigma_L^{+C}} Y_{2\Sigma} \boldsymbol{\ell}'_L + \phi_2^+ \overline{\Sigma_L^{0C}} Y_{2\Sigma} \boldsymbol{\ell}'_L) + h.c\Big] \tag{6.57}$$

In terms of mass eigenstates, this becomes

$$\begin{aligned}
\mathcal{L}_{Y,\Delta} = \frac{1}{v_2}\Big(&\sin\alpha \overline{N} V_{PMNS} M_N^{diag} V_{\ell N}^\dagger P_L \boldsymbol{\nu} h^0 + \cos\alpha \overline{N} V_{PMNS} M_N^{diag} V_{\ell N}^\dagger P_L \boldsymbol{\nu} H^0 \\
&+ i\cos\beta \overline{N} V_{PMNS} M_N^{diag} V_{\ell N}^\dagger P_L \boldsymbol{\nu} A^0 - 2\cos\beta \overline{\boldsymbol{\nu}} V_{PMNS}^T V_{\ell N}^* M_E^{diag} P_L E H^+ \\
&+ \sqrt{2}\sin\alpha \overline{E} M_E^{diag} V_{\ell N}^\dagger P_L \boldsymbol{\ell} h^0 + \sqrt{2}\cos\alpha \overline{E} M_E^{diag} V_{\ell N}^\dagger P_L \boldsymbol{\ell} H^0 \\
&+ \sqrt{2} i \cos\beta \overline{E} M_E^{diag} V_{\ell N}^\dagger P_L \boldsymbol{\ell} A^0 + \sqrt{2}\cos\beta \overline{N} M_N^{diag} V_{\ell N}^\dagger P_L \boldsymbol{\ell} H^+\Big) + h.c
\end{aligned} \tag{6.58}$$

Once again, the possible decays are much larger compared to the $E \to \ell h$ and $N \to \nu h$ decays that are possible in Type III see-saw+1HDM. The possibilities are

$$\begin{aligned}
&E \to \ell h^0/H^0/A^0 \\
&E^+ \to \overline{\nu} H^+, \quad E^- \to \nu H^- \\
&N \to \ell^\pm H^\mp/\nu h^0/\nu H^0/\nu A^0
\end{aligned} \tag{6.59}$$

I now continue with the simplified picture adopted in chapter 5 with only the lightest of the heavy entities, E_j and N_j and $m_\Sigma = m_E = m_N$ being the common triplet mass. I will write the parameters of this theory in terms of 2HDM parameters and those of the previous picture (Type III see-saw+1HDM). Distinguishing the $V_{\ell N}$'s of these two pictures as $V_{\ell N;1}$ and $V_{\ell N;2}$ so as to indicate the number of scalar doublets present, we can write

$$\frac{V_{\ell N;2}}{V_{\ell N;1}} = \frac{Y_{2\Sigma}^\dagger}{Y_{1\Sigma}^\dagger}.\frac{v_2}{v_1} = \left(\frac{Y_{2\Sigma}^\dagger}{Y_{1\Sigma}^\dagger}\right)\tan\beta \tag{6.60}$$

Using the value of M_W from eqn. (6.40) to write $v_2 = 2M_W\sin\beta/g$, the triplet-Yukawa interaction can be written as

$$\begin{aligned}
\mathcal{L}_{Y,\Delta} = \Big[&\frac{g m_\Sigma}{2M_W}\Big\{\frac{\sin\alpha}{\cos\beta}\overline{N} V_{\ell N;1}^\dagger \left(\frac{Y_{2\Sigma}^\dagger}{Y_{1\Sigma}^\dagger}\right) P_L \boldsymbol{\nu} h^0 + \frac{\cos\alpha}{\cos\beta}\overline{N} V_{\ell N;1}^\dagger \left(\frac{Y_{2\Sigma}^\dagger}{Y_{1\Sigma}^\dagger}\right) P_L \boldsymbol{\nu} H^0 \\
&+ i\overline{N} V_{\ell N;1}^\dagger \left(\frac{Y_{2\Sigma}^\dagger}{Y_{1\Sigma}^\dagger}\right) P_L \boldsymbol{\nu} A^0\Big\} \\
&+ \frac{g m_\Sigma}{\sqrt{2}M_W}\Big\{\frac{\sin\alpha}{\cos\beta}\overline{E} V_{\ell N;1}^\dagger \left(\frac{Y_{2\Sigma}^\dagger}{Y_{1\Sigma}^\dagger}\right) P_L \boldsymbol{\ell} h^0 + \frac{\cos\alpha}{\cos\beta}\overline{E} V_{\ell N;1}^\dagger \left(\frac{Y_{2\Sigma}^\dagger}{Y_{1\Sigma}^\dagger}\right) P_L \boldsymbol{\ell} H^0 \\
&+ i\overline{E} V_{\ell N;1}^\dagger \left(\frac{Y_{2\Sigma}^\dagger}{Y_{1\Sigma}^\dagger}\right) P_L \boldsymbol{\ell} A^0\Big\} \\
&- \frac{g m_\Sigma}{M_W}\Big\{\overline{\boldsymbol{\nu}} V_{\ell N;1}^* \left(\frac{Y_{2\Sigma}^\dagger}{Y_{1\Sigma}^\dagger}\right) P_L E H^+\Big\} + \frac{g m_\Sigma}{\sqrt{2}M_W}\Big\{\overline{N} V_{\ell N;1}^\dagger \left(\frac{Y_{2\Sigma}^\dagger}{Y_{1\Sigma}^\dagger}\right) P_L \boldsymbol{\ell} H^+\Big\} + h.c\Big]
\end{aligned} \tag{6.61}$$

The constraints on $V_{\ell N;1}$ are already mentioned in eqns. (5.54) and (5.55). However, as mentioned in the same section, these constraints are not so relevant for heavy triplet production at the LHC since they take place through gauge interactions of order unity.

6.3 Yukawa couplings of quarks

The Yukawa Lagrangian of u-type quarks in the present model is given by

$$\mathcal{L}_Y^U = -\Big(\overline{Q'_L} Y'^U \widetilde{\Phi_2} q'^U_R + \overline{q'^U_R} Y'^{\dagger U} \widetilde{\Phi_2^\dagger} Q'_L\Big) \tag{6.62}$$

and those of d-type quarks is given by

$$\mathcal{L}_Y^D = -\Big(\overline{Q'_L} Y'^D \Phi_1 q'^D_R + \overline{q'^D_R} Y'^{\dagger D} \Phi_1^\dagger Q'_L\Big) \tag{6.63}$$

The interactions of the physical Higgses with the quarks are presented below

$$\begin{aligned}
\mathcal{L}^{h^0}_{quarks} &= -\Big(\frac{\sin\alpha}{v_2}\overline{u}M^U u + \frac{\cos\alpha}{v_1}\overline{d}M^D d\Big)h^0 \\
\mathcal{L}^{H^0}_{quarks} &= -\Big(\frac{\cos\alpha}{v_2}\overline{u}M^U u - \frac{\sin\alpha}{v_1}\overline{d}M^D d\Big)H^0 \\
\mathcal{L}^{A^0}_{quarks} &= \Big(\frac{i\cos\beta}{v_2}\overline{u}M^U\gamma_5 u + \frac{i\sin\beta}{v_1}\overline{d}M^D\gamma_5 d\Big)A^0 \\
\mathcal{L}^{H^+}_{quarks} &= \Big(\frac{\sqrt{2}\cos\beta}{v_2}\overline{u}V_{CKM}M^U P_L d + \frac{\sqrt{2}\sin\beta}{v_1}\overline{u}V_{CKM}M^D P_R d\Big)H^+ \\
\mathcal{L}^{H^-}_{quarks} &= \Big(\frac{\sqrt{2}\cos\beta}{v_2}\overline{d}V^\dagger_{CKM}M^U P_R u + \frac{\sqrt{2}\sin\beta}{v_1}\overline{d}V^\dagger_{CKM}M^D P_L u\Big)H^-
\end{aligned} \tag{6.64}$$

where

$$u = \begin{pmatrix} u \\ c \\ t \end{pmatrix} \qquad d = \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{6.65}$$

represent the arrays of u and d-type quarks and

$$M^U = \frac{Y^U v_2}{\sqrt{2}}, \qquad M^D = \frac{Y^D v_1}{\sqrt{2}} \tag{6.66}$$

are the diagonalized mass matrices for the same.

	$E^+ \to \overline{\nu} W^+$	$E^+ \to \ell^+ Z$	$E^+ \to \ell^+ h^0$	$E^+ \to \ell^+ H^0$	$E^+ \to \ell^+ A^0$	$E^+ \to \overline{\nu} H^+$
$E^- \to \nu W^-$	$\nu\overline{\nu} W^+ W^-$	$\ell^+ \nu Z W^-$	$\ell^+ \nu h^0 W^-$	$\ell^+ \nu H^0 W^-$	$\ell^+ \nu A^0 W^-$	$\nu\overline{\nu} H^+ W^-$
$E^- \to \ell^- Z$	$\ell^- \overline{\nu} W^+ Z$	$\ell^+ \ell^- Z Z$	$\ell^+ \ell^- Z h^0$	$\ell^+ \ell^- Z H^0$	$\ell^+ \ell^- Z A^0$	$\ell^- \overline{\nu} H^+ Z$
$E^- \to \ell^- h^0$	$\ell^- \overline{\nu} h^0 W^+$	$\ell^+ \ell^- Z h^0$	$\ell^+ \ell^- h^0 h^0$	$\ell^+ \ell^- h^0 H^0$	$\ell^+ \ell^- A^0 h^0$	$\ell^- \overline{\nu} h^0 H^+$
$E^- \to \ell^- H^0$	$\ell^- \overline{\nu} H^0 W^+$	$\ell^+ \ell^- H^0 Z$	$\ell^+ \ell^- h^0 H^0$	$\ell^+ \ell^- H^0 H^0$	$\ell^+ \ell^- H^0 A^0$	$\ell^- \overline{\nu} H^0 H^+$
$E^- \to \ell^- A^0$	$\ell^- \overline{\nu} A^0 W^+$	$\ell^+ \ell^- A^0 Z$	$\ell^+ \ell^- h^0 A^0$	$\ell^+ \ell^- A^0 H^0$	$\ell^+ \ell^- A^0 A^0$	$\ell^- \overline{\nu} A^0 H^+$
$E^- \to \nu H^-$	$\nu\overline{\nu} W^+ H^-$	$\nu \ell^+ H^- Z$	$\ell^+ \nu h^0 H^-$	$\ell^+ \nu H^0 H^-$	$\ell^+ \nu A^0 H^-$	$\nu\overline{\nu} H^- H^+$

Table 6: Final states for E^+E^- production in Type III see-saw+Type II 2HDM.

The number of possible final states in this case is much larger due to the increased decay channels offered by the wider particle spectrum. I tabulate the possible final states for the decays in production channels under consideration without including the decays of the gauge bosons.

	$E^+ \to \overline{\nu} W^+$	$E^+ \to \ell^+ Z$	$E^+ \to \ell^+ h^0$	$E^+ \to \ell^+ H^0$	$E^+ \to \ell^+ A^0$	$E^+ \to \overline{\nu} H^+$
$N \to \ell^- W^+$	$\ell^- \overline{\nu} W^+ W^+$	$\ell^+ \ell^- W^+ Z$	$\ell^+ \ell^- \ell^- h^0 W^+$	$\ell^+ \ell^- H^0 W^+$	$\ell^+ \ell^- A^0 W^+$	$\ell^- \overline{\nu} W^+ H^+$
$N \to \ell^+ W^-$	$\ell^+ \overline{\nu} W^- W^+$	$\ell^+ \ell^- Z W^-$	$\ell^+ \ell^- h^0 W^-$	$\ell^+ \ell^- W^- H^0$	$\ell^+ \ell^- W^- A^0$	$\ell^+ \overline{\nu} W^- H^+$
$N \to \nu Z$	$\nu \overline{\nu} Z W^+$	$\ell^+ \nu Z Z$	$\ell^+ \nu Z h^0$	$\ell^+ \nu Z H^0$	$\ell^+ \nu Z A^0$	$\nu \overline{\nu} Z H^+$
$N \to \nu h^0$	$\nu \overline{\nu} h^0 W^+$	$\ell^+ \nu h^0 Z$	$\ell^+ \nu h^0 h^0$	$\ell^+ \nu h^0 H^0$	$\ell^+ \nu h^0 A^0$	$\nu \overline{\nu} h^0 H^+$
$N \to \nu H^0$	$\nu \overline{\nu} H^0 W^+$	$\ell^+ \nu H^0 Z$	$\ell^+ \nu h^0 H^0$	$\ell^+ \nu H^0 H^0$	$\ell^+ \nu H^0 A^0$	$\nu \overline{\nu} H^0 H^+$
$N \to \nu A^0$	$\nu \overline{\nu} A^0 W^+$	$\ell^+ \nu A^0 Z$	$\ell^+ \nu A^0 h^0$	$\ell^+ \nu A^0 H^0$	$\ell^+ \nu A^0 A^0$	$\nu \overline{\nu} A^0 H^+$
$N \to \ell^- H^+$	$\ell^- \overline{\nu} H^+ W^+$	$\ell^+ \ell^- H^+ Z$	$\ell^+ \ell^- h^0 H^+$	$\ell^+ \ell^- H^0 H^+$	$\ell^+ \ell^- A^0 H^+$	$\ell^- \overline{\nu} H^+ H^+$
$N \to \ell^+ H^-$	$\ell^+ \overline{\nu} H^- W^+$	$\ell^+ \ell^+ H^- Z$	$\ell^+ \ell^+ h^0 H^-$	$\ell^+ \ell^+ H^0 H^-$	$\ell^+ \ell^+ A^0 H^-$	$\ell^+ \overline{\nu} H^- H^+$

Table 7: Final states for $E^{\pm} N$ production in Type III see-saw+Type II 2HDM.

Chapter 7

Summary and Conclusion

In this thesis work, I have addressed the fascinating issue of the observed smallness of neutrino masses by means of the see-saw mechanisms viz. Types I, II and III. The content of the work can be summarized in the following paragraphs

- In the introductory part of Chapter 1, I explain why the Standard Model fails to accommodate massive neutrinos. Then I present the important concept of Majorana neutrino mass and how the neutrino can qualify as a Majorana fermion. The consequences of incorporating right-handed neutrinos in the Standard Model are next dealt with and the chapter concludes with some properties of Majorana masses.

- The theory of the Type I see-saw mechanism is explained in detail in chapter 2. Beginning with a simple one generation Dirac-Majorana neutrino mixing picture, I show explicitly how heavy fields can be integrated out to arrive at an effective theory with the dim-5 operator $\mathcal{L}_5$ which generates small Majorana masses for left-handed neutrinos. The generalized picture with N_s right-handed neutrinos is then worked out and the interactions with the Standard Model gauge bosons W and Z are shown.

- Chapter 3 deals with extending the scalar sector of the Standard Model by adding an $SU(2)_L$ scalar triplet Δ, or the so-called Type II see-saw mechanism. Special care is devoted to the charged lepton couplings with the scalar triplet and interesting possibilities to probe the flavor Yukawa structure through like-sign dilepton decays are explained. Electroweak precision data on the ρ-parameter is used to constrain the triplet VEV, v_t and order of magnitudes of various parameters in the neutrino mass matrix are tabulated.

- Chapter 4 is an exposition on the Type III see-saw mechanism in which atleast two heavy $SU(2)_L$ fermion triplets are added. The structure of the light neutrino mass matrix is quite similar to that in the Type I see-saw picture with the added feature of mixing between light and heavy charged leptons. A comprehensive analysis of the various interactions of the fermion triplets and possible decays of the heavy fermions are also discussed.

- In chapter 5, I make a brief transition from theory to phenomenology in that I review the test possibilities of see-saw mechanisms at the CERN LHC based on simulation studies. The signal types for each of the see-saw mechanisms are systematically classified depending on the lepton multiplicity of the final state. Potential advantages and drawbacks of each of them are also discussed in detail. The trilepton final state seems promising for early discoveries and can act as the 'golden channel' for indirect searches of the heavy mediators involved in the see-saw mechanisms.

- In the concluding chapter, I use the tools developed in the previous chapters to construct a model which is an admixture of Type III see-saw mechanism and 2HDM. I motivate the imposition of a discrete Z_2 symmetry and then softly break it with appropriate justifications towards such a predilection. The gauge interactions of the Higgs doublets as well as the heavy fermion triplets are worked out in detail with charge assignments in accordance with the Type II 2HDM. A correlation with the Yukawa interactions in the pure Type III see-saw and important 2HDM parameters is also made. The discussion concludes with the Yukawa interactions of u and d-type quarks with the scalar doublets.

The field of neutrino physics is increasingly becoming a popular field of research with overlaps in many other regimes of physics like cosmology, astro(particle)physics and Supersymmetry (SUSY) phenomenology. The discovery of positive signatures of the see-saw mechanisms at high energy accelerators in the coming future will lead to a concrete footing of the theory of neutrino masses. In the world of particle physics which delights us every time with more and more surprising discoveries, it would not be skeptical to say that such possibilities are only separated by time. The LHC at CERN is reawakening for its second run at 13 TeV even as I am writing this. Hopefully, it would surprise us with many unforeseen discoveries, sort out existing possibilities and bring us one step closer to the realm of truth.

Appendix A

Diagonalization of a general mass matrix

A symmetric complex $N \times N$ matrix M^M can be diagonalized by the transformation

$$(V_L^\nu)^T M^M V_L^\nu = M, \qquad M_{kj} = m_k \delta_{kj} \quad \text{(k,j=1,2,....,N)} \tag{A.1}$$

where V_L^ν is a unitary matrix and m_k are real and postive masses. To justify that the matrix M^M can be diagonalized, let us look at the number of available parameters for diagonalization. The $N \times N$ symmetric complex matrix M^M is determined by $N(N+1)$ independent real parameters, which is equal to the N^2 independent real parameters of the $N \times N$ unitary matrix V_L^ν plus the N independent real elements of the diagonal $N \times N$ matrix M. So, the number of available parameters is sufficient.
To prove the diagonalization, we use the fact that an arbitrary complex matrix can be diagonalized by the biunitary transformation

$$V^\dagger M^M W = M, \qquad with \qquad M_{kj} = m_k \delta_{kj} \quad \text{(k,j=1,2,....,N)} \tag{A.2}$$

where V and W are unitary matrices. Therefore

$$M^M = VMW^\dagger \tag{A.3}$$

from which we have

$$M^M (M^M)^\dagger = VMW^\dagger W M^\dagger V^\dagger = VM^2V^\dagger \tag{A.4}$$

from which we have

$$(M^M)^T((M^M)^T)^\dagger = (W^\dagger)^T M V^T (V^T)^\dagger M^\dagger W^T = (W^\dagger)^T M^2 W^T \tag{A.5}$$

Since M^M is symmetric, the two expressions are equivalent.

$$VM^2V^\dagger = (W^\dagger)^T M^2 W^T \tag{A.6}$$

Multiplying on the left by W^T and on the right by V, we see that

$$[W^T V, M^2] = 0 \tag{A.7}$$

Now, $W^T V$ being unitary and commutating with a diagonal matrix, it must itself be a diagonal matrix of phases.

$$W^T V = D, \qquad D_{kj} = e^{i\varphi_k}\delta_{kj} \tag{A.8}$$

So,

$$\begin{aligned} M^M = VMW^\dagger &= (W^\dagger)^T D M W^\dagger \\ &= (D^{1/2}W^\dagger)^T M (D^{1/2}W^\dagger) \end{aligned} \tag{A.9}$$

So, $V_L^{\nu\dagger} = D^{1/2}W^\dagger$ or $V_L^\nu = WD^{-1/2}$. Thus, the diagonalizing matrix is the product of a unitary matrix and a diagonal matrix of phases.

Appendix B

Diagonalization of charged lepton and neutrino mass matrices in the type III see-saw model

The diagonalization of the charged lepton and neutrino mass matrices are carried out using unitary matrices $U^{L,R}$ and U^0 as

$$U^{R\dagger} \begin{pmatrix} m_\ell & 0 \\ Y_\Sigma v & M_\Sigma \end{pmatrix} U^L = \begin{pmatrix} m^\ell_{light} & 0 \\ 0 & m^\ell_{heavy} \end{pmatrix} \tag{B.1}$$

$$U^{0^T} \begin{pmatrix} 0 & \frac{Y_\Sigma^T v}{\sqrt{2}} \\ \frac{Y_\Sigma v}{\sqrt{2}} & M_\Sigma \end{pmatrix} U^0 = \begin{pmatrix} m^\nu_{light} & 0 \\ 0 & m^\nu_{heavy} \end{pmatrix} \tag{B.2}$$

In the see-saw approximation whereby M_Σ is large, we can diagonalize the mass matrices by following the ansatz mentioned in eqn. (2.43).

Let $U^0 = e^{iH} V$, where

$$H = \begin{pmatrix} 0 & S_1 \\ S_2 & 0 \end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix} V_1 & 0 \\ 0 & V_2 \end{pmatrix} \tag{B.3}$$

are unitary matrices with $S_{1,2}$ of $\mathcal{O}(Y_\Sigma v M_\Sigma^{-1})$ and $V_{1,2}$ of $\mathcal{O}(1)$ respectively.

Writing the leading order expressions up to $Y_\Sigma^2 v^2 M_\Sigma^2$, the form of U^0 becomes

$$U^0 \simeq \begin{pmatrix} 1-\frac{1}{2}S_1S_2 & iS_1 \\ iS_2 & 1-\frac{1}{2}S_2S_1 \end{pmatrix} \tag{B.4}$$

Then from the condition that the off-diagonal elements in the left hand side of eqn. (B.1) must vanish, we can write

$$Y_\Sigma^T \frac{v}{\sqrt{2}} - \frac{v}{2\sqrt{2}} v S_2 S_1 - \frac{v}{2\sqrt{2}} S_2^T S_1^T Y_\Sigma^T + \frac{v}{4\sqrt{2}} S_2^T S_1^T Y_\Sigma^T S_2 S_1 - \frac{v}{\sqrt{2}} S_2^T Y_\Sigma S_1 + i S_2^T M_\Sigma - \frac{i}{2} S_2^T M_\Sigma S_2 S_1 = 0 \tag{B.5}$$

and

$$-v S_1^T Y_\Sigma^T S_2 + \frac{v}{\sqrt{2}} Y_\Sigma - \frac{v}{2\sqrt{2}} Y_\Sigma S_1 S_2 - i M_\Sigma S_2 - \frac{v}{2\sqrt{2}} S_1^T S_2^T Y_\Sigma + \frac{1}{4\sqrt{2}} S_1^T S_2^T Y_\Sigma S_1 S_2 + \frac{i}{2} S_1^T S_2^T M_\Sigma S_2 = 0 \tag{B.6}$$

Since we are interested only in terms to leading order in $Y_\Sigma v M_\Sigma^{-1}$, so the condition in eqn. (B.5) simplifies to

$$Y_\Sigma^T \frac{v}{\sqrt{2}} + i S_2^T M_\Sigma = 0 \quad \text{or} \quad S_2 = \frac{iv}{\sqrt{2}} M_\Sigma^{-1} Y_\Sigma \tag{B.7}$$

From the unitarity of H, we obtain $S_1 = S_2^\dagger = -iv Y_\Sigma^\dagger M_\Sigma^{-1}/\sqrt{2}$ and hence the diagonalizing matrix becomes

$$\begin{aligned} U^0_{\nu\nu} &= (1-\epsilon/2) V_{PMNS}, \quad U^0_{\nu N} = Y_\Sigma^\dagger M_\Sigma^{-1} v/\sqrt{2}, \\ U^0_{N\nu} &= -M_\Sigma^{-1} Y_\Sigma U^0_{\nu\nu} v/\sqrt{2}, \quad U^0_{NN} = 1-\epsilon'/2 \end{aligned} \tag{B.8}$$

where $\epsilon = S_1 S_2 = Y_\Sigma^\dagger M_\Sigma^{-2} Y_\Sigma v^2/2$ and $\epsilon' = S_2 S_1 = M_\Sigma^{-1} Y_\Sigma Y_\Sigma^\dagger M_\Sigma^{-1} v^2/2$. The V_{PMNS} arises because we are working in the basis where the light neutrino mass matrix is diagonal.

For the charged lepton case, there are two different diagonalizing matrices U^L and U^R. Let U^L has the same form as U^0 but replace the matrices by their primed ones in case of U^R.

$$U^R \simeq \begin{pmatrix} 1-\frac{1}{2}S_1'S_2' & iS_1' \\ iS_2' & 1-\frac{1}{2}S_2'S_1' \end{pmatrix} \tag{B.9}$$

The vanishing of the off-diagonal elements in this case becomes

$$i m_\ell S_1 - \frac{i}{2} S_2'^\dagger S_1'^\dagger m_\ell S_1 + S_2'^\dagger Y_\Sigma v S_1 - i S_2'^\dagger M_\Sigma + \frac{i}{2} S_2'^\dagger M_\Sigma S_2 S_1 = 0 \tag{B.10}$$

and

$$-iS_1'^{\dagger}m_\ell + \frac{i}{2}S_1'^{\dagger}m_\ell S_1 S_2 + Y_\Sigma v - \frac{v}{2}Y_\Sigma S_1 S_2 + iM_\Sigma S_2 - \frac{v}{2}S_1'^{\dagger}S_2'^{\dagger}Y_\Sigma \\ + \frac{v}{4}S_1'^{\dagger}S_2'^{\dagger}Y_\Sigma S_1 S_2 - \frac{i}{2}S_1'^{\dagger}S_2'^{\dagger}M_\Sigma S_2 = 0 \tag{B.11}$$

Again, since we are interested in terms of leading order in $Y_\Sigma v M_\Sigma^{-1}$, so the above conditions reduce to

$$S_1' = m_\ell S_1 M_\Sigma^{-1} \quad \text{and} \quad S_2 = iM_\Sigma^{-1}Y_\Sigma v \tag{B.12}$$

so that $S_1 = -ivY_\Sigma^{\dagger}M_\Sigma^{-1}$, $S_1' = -ivm_\ell Y_\Sigma^{\dagger}M_\Sigma^{-2}$ and $S_2' = -ivM_\Sigma^{-2}Y_\Sigma m_\ell$.

The diagonalizing matrices U^L and U^R can now be written in terms of the block matrices as

$$U^L_{\ell\ell} = 1-\epsilon, \quad U^L_{\ell E} = Y_\Sigma^{\dagger}M_\Sigma^{-1}v, \quad U^L_{E\ell} = -M_\Sigma^{-1}Y_\Sigma v, \quad U^L_{EE} = 1-\epsilon', \\ U^R_{\ell\ell} = 1, \quad U^R_{\ell E} = m_\ell Y_\Sigma^{\dagger}M_\Sigma^{-2}v, \quad U^R_{E\ell} = -M_\Sigma^{-2}Y_\Sigma m_\ell v, \quad U^R_{EE} = 1 \tag{B.13}$$

where ϵ and ϵ' are already defined.

Appendix C

Theoretical constraints on the 2HDM parameters

The parameters of Higgs potential in 2HDM are constrained by

- Vacuum stability: This condition implies that the potential remains positive at large quasi-classical values of fields $\mid \phi_k \mid$. For soft Z_2 violation, this translates to [49]

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0, \quad \lambda_3 + \lambda_4 - \mid \lambda_5 \mid + \sqrt{\lambda_1 \lambda_2} > 0 \tag{C.1}$$

- Minimum constraints: This requires that the minimum of the potential with $v = \sqrt{v_1^2 + v_2^2} = 246$ GeV is the global minimum which leads to [50]

$$m_{12}^2 (m_{11}^2 - m_{22}^2 \sqrt{\lambda_1/\lambda_2})(\tan\beta - (\lambda_1/\lambda_2)^{1/4}) > 0 \tag{C.2}$$

- Perturbativity constraints: For perturbative calculations to be trustworthy, the λ_i's should not be too large. A simple approach is requiring $\mid \lambda_i \mid < \lambda_{max}$ and some $\lambda_{max} > 0$ [44]. The effective parameters of perturbation theory for Yukawa interactions is $g^2/(4\pi)^2$ which implies that a necessary condition for the smallness of radiative corrections is $\mid g \mid << 4\pi$. Hence, a conservative choice is $\lambda_{max} = 4\pi$.

Bibliography

[1] W. Pauli, A letter to L. Meitner and her colleagues; L. M. Brown, "*The Idea of the Neutrino*", Physics Today **31** (1978) 9.

[2] T. Kajita (Kamiokande and Super-Kamiokande Collaborations), Int. Conf. on Neutrino Physics and Astrophysics (Neutrino 98);
T. Kajita (Kamiokande and Super-Kamiokande Collaborations), *Atmospheric neutrino results from Super-Kamiokande and Kamiokande — Evidence for $\nu\mu$ oscillations*, Nucl. Phys. Proc. Supp. **77** (1999) 123.

[3] Y. Fukuda et. al. (Super-Kamiokande Collaboration), *Evidence for Oscillation of Atmospheric Neutrinos*, Phys. Rev. Lett. **81** (1998) 1562.

[4] E. Majorana, *Teoria Simmetrica dell electtrone del positrone*, Nuovo Cimento, **14** (1937) 171; English Translation by Luciano Maiani, *A symmetric theory of electrons and positrons*, Ettore Majorana Scientific Papers (2006) 201.

[5] P. Minkowski, Phys. Lett. B **67**, 421 (1977); T. Yanagida, in Workshop on Unified Theories, KEK report 79-18 p.95 (1979); M. Gell-Mann, P. Ramond, R. Slansky, in Supergravity (North Holland, Amsterdam, 1979) eds. P. van Nieuwenhuizen, D. Freedman, p.315; S. L. Glashow, in 1979 Cargese Summer Institute on Quarks and Leptons (Plenum Press, New York, 1980) eds. M. Levy, J. L. Basdevant, D. Speiser, J. Weyers, R. Gastmans and M. Jacobs, p.687; R. Barbieri, D. V. Nanopoulos, G. Morchio and F. Strocchi, Phys. Lett. B 90, **91** (1980); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980); G. Lazarides, Q. Shafi and C. Wetterich, Nucl. Phys. B **181**, (1981) 287.

[6] J. Schechter and J. W. F. Valle, *Neutrino decay and spontaneous violation of lepton number*, Phys. Rev. D ,**25** (1982) 774.

[7] Boris Kayser and Robert Schrock, *Distinguishing between Dirac and Majorana neutrinos in neutral-current reactions*, Phys. Lett. B, **112** (1982) 137.

[8] R. N. Mohapatra, G. Senjanovic, Phys. Rev. D **23** (1981) 165; T.P. Cheng, L. F. Li, Phys. Rev. D **22** (1980) 2860; G. Lazarides, Q. Shafi, C. Wetterich, Nucl. Phys. B **181** (1981) 287-300; M. Magg, C. Wetterich, Phys. Lett. B **94** (1980) 61.

[9] W. Konetschny and W. Kummer, *Nonconservation of total lepton number with scalar bosons*, Phys. Lett. B **70** (1977) 433.

[10] G. B. Gelmini and M. Roncadelli, *Left-handed neutrino mass scale and spontaneously broken lepton number*, Phys. Lett. B **99** (1981) 411.

[11] A. Arhrib, R. Benbrik, M. Chabab, G. Moultaka, M. C. Peyranère, L. Rahili, and J. Ramadan, *The Higgs Potential in the Type II Seesaw Model*, Phys. Rev. D **84** (2011), 095005; arXiv:1105.1925 [hep-ph].

[12] Pavel Fileviez Perez, Tao Han, Gui Yu Huang, Tong Li, Kai Wang, *Neutrino Masses and the LHC: Testing Type II Seesaw*, Phys. Rev. D **78** (2008) 015018; arXiv:0805.3536 [hep-ph].

[13] A. G. Akeroyd, Mayumi Aoki, and Hiroaki Sugiyama, *Probing Majorana Phases and Neutrino Mass Spectrum in the Higgs Triplet Model at the LHC*, Phys. Rev. D **77**(2008), 075010; arXiv:0712.4019 [hep-ph].

[14] Julia Garayoa and Thomas Schwetz, *Neutrino mass hierarchy and Majorana CP phases within the Higgs triplet model at the LHC*, JHEP03(2008)009; arXiv:0712.1453 [hep-ph].

[15] Alejandra Melfo, Miha Nemevsek, Fabrizio Nesti, Goran Senjanovic, Yue Zhang, *Type II Seesaw at LHC: the Roadmap*, Phys.Rev. D **85** (2012) 055018; arXiv:1108.4416 [hep-ph].

[16] Claude Amsler et al., Review of particle physics, Phys. Lett. B **667** (2008), 1-1340.

[17] R. Foot, H. Lew, X. G. He and G.C. Joshi, *Seesaw Neutrino Masses Induced by a Triplet of Leptons*, Z. Phys. C **44** (1989) 441.

[18] Tong Li, Xiao-Gang He, *Neutrino Masses and Heavy Triplet Leptons at the LHC: Testability of Type III Seesaw*, Phys.Rev. D **80** (2009) 093003; arXiv:0907.4193 [hep-ph].

[19] A. Abada, C.Biggio, F. Bonnet, M.B. Gavela, T. Hambye, *Low energy effects of neutrino masses*, JHEP 0712:061 (2007); arXiv:0707.4058 [hep-ph].

[20] F. del Aguila, J.A. Aguilar-Saavedra, J. de Blas, *Trilepton signals: the golden channel for seesaw searches at LHC*, Acta Phys. Polon. B **40** (2009), 2901-2911; arXiv:0910.2720 [hep-ph].

[21] G. Aad et al. [ATLAS Collaboration], *Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC*, Phys. Lett. B **716** (2012) 1; arXiv:1207.7214 [hep-ex].

[22] ATLAS Collaboration, G. Aad et al., *Combined measurements of the mass and signal strength of the Higgs-like boson with the ATLAS detector using up to 25 fb*$^{-1}$ *of proton-proton collision data*, Tech. Rep. ATLAS-CONF-2013-14.

[23] S. Chatrchyan et al. [CMS Collaboration], *Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC*, Phys. Lett. B **716** (2012) 30; arXiv:1207.7235 [hep-ex].

[24] CMS Collaboration, S. Chatrchyan et al., *Combination of standard model Higgs boson searches and measurements of the properties of the new boson with a mass near 125 GeV*, Tech. Rep. CMS-PAS-HIG-13-005.

[25] W. Y. Keung and G. Senjanovic, *Majorana Neutrinos and the Production of the Right-Handed Charged Gauge Boson*, Phys. Rev. Lett. **50** (1983) 1427.

[26] A. Datta, M. Guchait and D. P. Roy, *Prospect of Heavy Right-Handed Neutrino Search at SSC/LHC Energies*, Phys. Rev. D **47** (1993) 961; [hep-ph/9208228].

[27] A. Ferrari et. al. , *Sensitivity study for new gauge bosons and right-handed Majorana neutrinos in pp collisions at* $\sqrt{s} = 14\ TeV$, Phys. Rev. D **62** (2000) 013001.

[28] S. N. Gninenko, M. M. Kirsanov, N. V. Krasnikov and V. A. Matveev, *Detection of heavy Majorana neutrinos and right-handed bosons*, Phys. Atom. Nul. **70** (2007) 441.

[29] F. del Aguila and J. A. Aguilar-Saavedra, *Like-sign dilepton signals from a leptophobic Z' boson*, JHEP 0711 (2007) 072; arXiv:0705.4117 [hep-ph].

[30] K. Huitu, S. Khalil, H. Okada and S. K. Rai, *Signatures for right-handed neutrinos at the Large Hadron Collider*, arXiv:0803.2799 [hep-ph].

[31] M. Hirsh, M. A. Diaz, W. Porod, J. C. Romao and J. W. F. Valle, *Neutrino Masses and Mixings from Supersymmetry with Bilinear R–Parity Violation: A Theory for Solar and Atmospheric Neutrino Oscillations*, Phys. Rev. D **62** (2000) 113008; [hep-ph/0004115].

[32] M. Hirsh and W. Porod, *Neutrino properties and the decay of the lightest supersymmetric particle*, Phys. Rev. D **68** (2003) 115007 [hep-ph/0307364].

[33] M. Hirsh, A. Viente and W. Porod, *Spontaneous R-parity violation: Lightest neutralino decays and neutrino mixing angles at future colliders*, Phys. Rev. D **77** (2008) 075005 ; arXiv:0802.2896 [hep-ph].

[34] F. del Aguila, J. de Blas and M. Perez-Vitoria, *Effects of new leptons in Electroweak Precision Data*, Phys. Rev. D **78** (2008) 013010; arXiv:0803.4008 [hep-ph].

[35] D. Tommasini, G. Barenboim, J. Bernabeu and C. Jarlskog, *Non-decoupling of Heavy Neutrinos and Lepton Flavour Violation*, Nucl. Phys. B **444** (1995) 451; [hep-ph/9503228].

[36] J. G. Korner, A. Pilaftsis and K. Shilher, *Leptonic Flavor-changing Z_0 Decays in $SU(2) \times U(1)$ Theories with Right-handed Neutrinos*, Phys. Lett. B **300** (1993) 381; [hep-ph/9301290].

[37] A. Ilakova and A. Pilaftsis, *Flavour-Violating Charged Lepton Decays in Seesaw-Type Models*, Nucl. Phys. B **437** (1995) 491 [hep-ph/9403398].

[38] H. V. Klapdor-Kleingrothaus et al., *Latest results from the Heidelberg-Moscow double beta decay experiment*, Eur. Phys. J. A **12** (2001) 147-154.

[39] F. del Aguila, J. A. Aguilar-Saavedra, *Distinguishing seesaw models at LHC with multi-lepton signals*, Nucl. Phys. B **813** (2009) 22-90; arXiv:0808.2468 [hep-ph].

[40] M. Cirelli, N. Fornengo and A. Strumia, *Minimal Dark Matter*, Nucl. Phys. B **753** (2006) 178; arXiv:hep-ph/0512090.

[41] A. Abada, C. Biggio, F. Bonnet, M. B. Gavela and T. Hambye, *$\mu \to e\gamma$ and $\tau \to \ell\gamma$ decays in the fermion triplet seesaw model*, Phys. Rev. D **78**, 033007, (2008); arXiv:0803.0481[hep-ph].

[42] A. G. Akeroyd and M. Aoki, *Single and Pair Production of Doubly Charged Higgs Bosons at Hadron Colliders*, Phys. Rev. D **72** (2005) 035011; hep-ph/0506176.

[43] For a review, see G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, Marc Sher, Joao P. Silva, *Theory and Phenomenology of two Higgs-doublet models*, Physics Reports **516** (2012), 1–102; arXiv:1106.0034 [hep-ph].

[44] Ilya F. Ginzburg, Maria Krawczyk, *Symmetries of Two Higgs Doublet Model and CP violation*, Phys. Rev. **D 72**, 115013 (2005); arXiv:hep-ph/0408011.

[45] Marcela Carena, Howard E. Haber, *Higgs Boson Theory and Phenomenology*, Prog. Part. Nucl. Phys **50** (2003), 63-152 ; arXiv:hep-ph/0208209.

[46] John F. Gunion, Howard E. Haber, *The CP-conserving two-Higgs-doublet model: the approach to the decoupling limit*, Phys. Rev. D **67** (2003) 075019; arXiv:hep-ph/0207010.

[47] Howard E. Haber, *Non-Minimal Higgs Sectors: The Decoupling Limit and its Phenomenological Implications*; arXiv:hep-ph/9501320.

[48] S. Kanemura, T. Kubota, H.-A. Tohyama, *Generalized screening theorem for Higgs decay processes in the two-doublet model*, Nucl. Phys. B **483** (1997) 111-140; arXiv:hep-ph/9604381.

[49] Boris Kastening, *Bounds from Stability and Symmetry Breaking on Parameters in the Two-Higgs-Doublet Potential*; arXiv:hep-ph/9307224.

[50] A. Barroso, P. M. Ferreira, I. P. Ivanov, Rui Santos, *Metastability bounds on the two Higgs doublet model*, arXiv:1303.5098 [hep-ph].